Chapter 15 Production

Production Problem: How a firm employs inputs (labor L and capital K) to
1. Maximize output Q for a given cost level C_0.
2. Minimize cost C to produce a give level of output Q_0.

- The understanding of this production problem is crucial to understanding the behavior of a firm in perfect competition and monopoly—and other markets to be discussed in EE311.

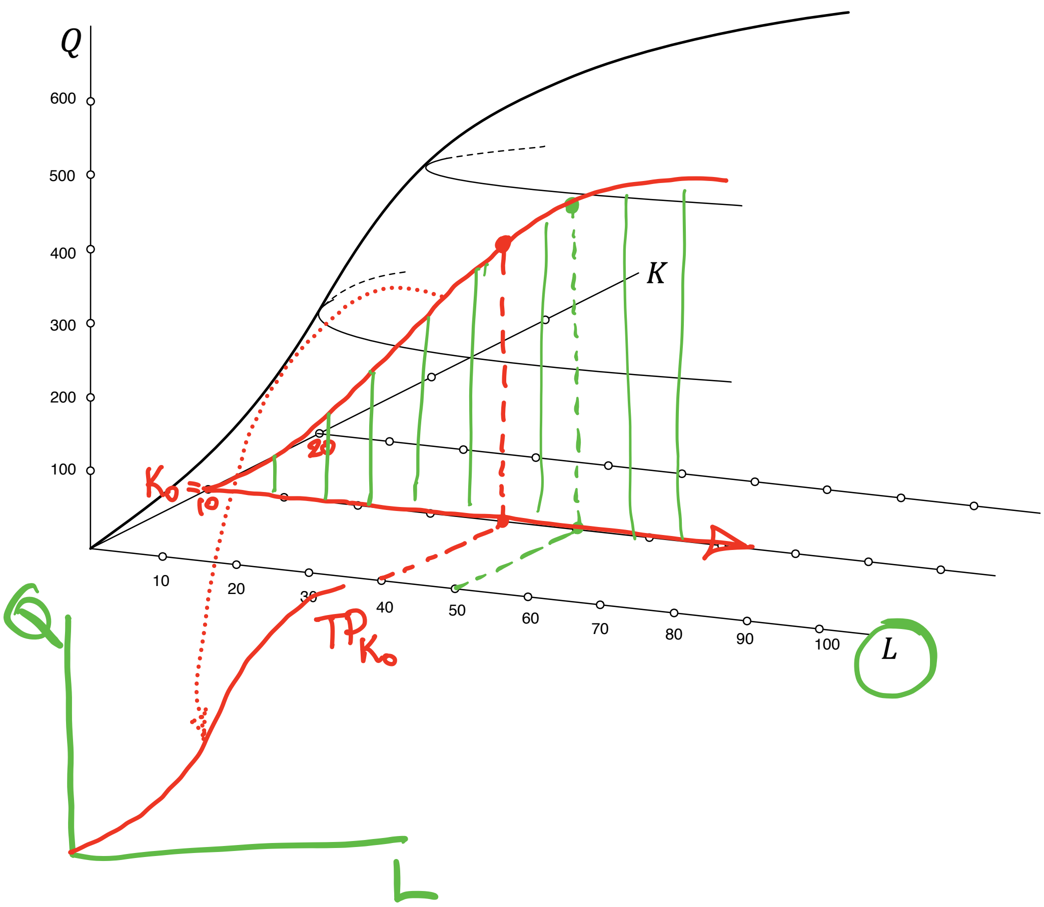
Production Function $Q = f(L, K)$ is a relationship between the highest output Q achievable from using labor $= L$ and capital $= K$ as efficiently as possible with the given technology

Example: $Q = f(L, K) = 10L^{0.5}K^{0.5}$,

$$\left.\begin{matrix} L = 400 \\ K = 900 \end{matrix}\right\} \Longrightarrow Q = 6{,}000 \ units$$

$$\left.\begin{matrix} L = 100 \\ K = 900 \end{matrix}\right\} \Longrightarrow Q = 3{,}000 \ units$$

Graph of a Production Function in 3D



Short-Run Production: Time frame that there is at least ***one input*** that cannot be changed (fixed factor)

- Usually in Short-Run, capital is assumed to be the fixed factor at $K = K_0$
- Fixed factor does not change with the quantity Q $\Longrightarrow$ Fixed Cost.

Long-Run Production: Time frame that is long enough for the firm to vary every input
$\Longrightarrow$ no fixed factor $\Longrightarrow$ no fixed cost.

<u>Short-Run Production</u> With production function

$$Q = f(L, K) = 10L^{0.5}K^{0.5}$$

Assume capital K is fixed at $K_0 = 900$, we have

$$Q = 10L^{0.5}K_0^{0.5}$$

$900 = 30$

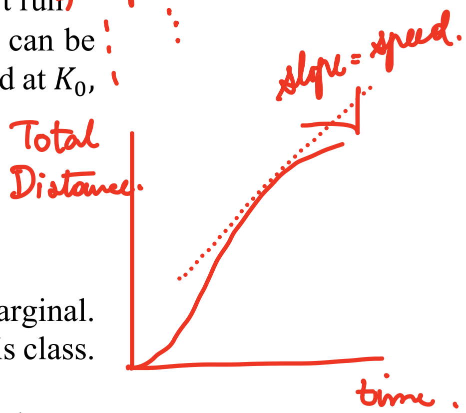
$$TP_{K_0}(L) = 300L^{0.5}$$

100

Q	L	$(K = K_0 = 300)$
0	0	
3000	100	
6,000	400	
⋮	⋮	

- we can change output Q by changing L in the short run
- This is called the Total Product is the output that can be produced at various labor level L, with capital fixed at K_0,

$$\boxed{TP_{K_0}(L) = f(L, K_0)}$$



Total, Average, and Marginal

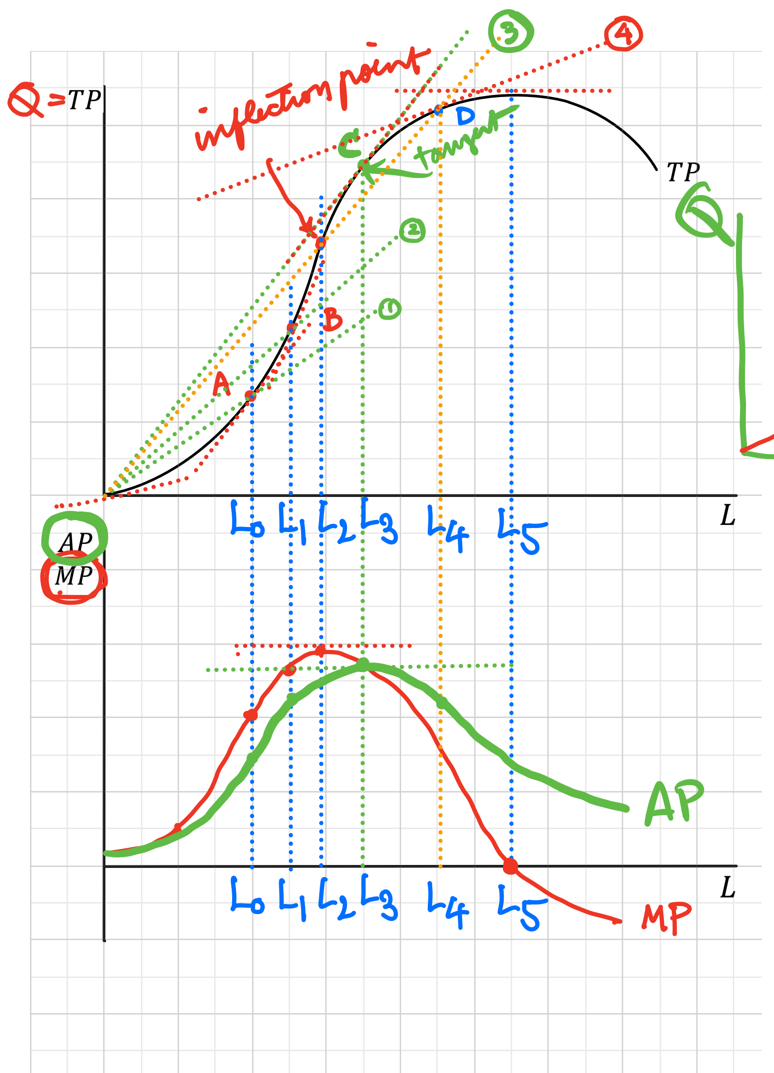
Given a Total, we can find Average and Marginal. This is the first T-A-M relation to be discussed in this class.

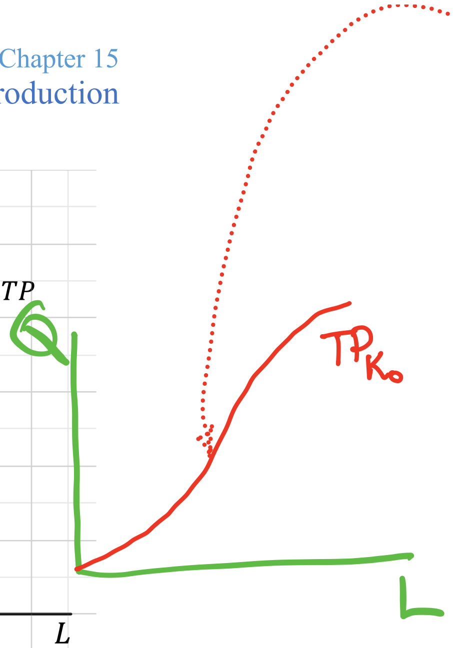
Given the Total Product $TP(L)$, (with subscript K_0 suppressed)

- Average Product $AP(L) = \frac{TP(L)}{L}$,
- Marginal Product $MP(L) = \boxed{\frac{dTP(L)}{dL}}$,

Marginal Product $MP(L)$ is the rate of change of the output $TP(L)$. That is, $MP(L)$ is the slope of $TP(L)$

The relationship of TP, AP and MP is demonstrated by the following graphs.





$L=5 \Rightarrow Q=100 \Rightarrow AP=\frac{100}{5}=20$

6^{th} person $MP=25$.

$L=6 \quad Q=125, AP=\frac{125}{6}=2.8$

Total	Average	Marginal	
	A increasing	M > A	< L_3
	A decreasing	M < A	> L_3
	A max	M = A.	(L_3)
T max		M = 0	(L_5)
T inflection		M max	(L_2)

- The relationship between Average and Marginal can also be verified by calculus. By definition,

$$AP = \frac{TP}{L}$$

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$MP(L) = \frac{d}{dL}TP(L) = \frac{d}{dL}(AP(L)\cdot L)$$

$$= AP(L)\cancel{\frac{dL}{dL}} + L\underbrace{\frac{dAP(L)}{dL}}_{\text{slope of AP}}$$

when $AP(L)$ is at max pt, slope of $AP = 0$:-

so $MP = AP$.

when slope $AP > 0$; $MP > AP$