

Solution: Quiz 4

- (a) (7 Points) Find the solution set for
- A
- that satisfies

$$\frac{A^2 - 5A}{A - 9} \leq \frac{4}{9 - A}.$$

- (b) (3 Points) Use (a) to find the solution set for
- x
- that satisfies

$$\frac{x^4 - 5x^2}{x^2 - 9} \leq \frac{4}{9 - x^2}.$$

Solution:

- (a) First note that
- $\frac{A^2 - 5A}{A - 9} \leq \frac{4}{9 - A}$
- is equivalent to
- $\frac{A^2 - 5A}{A - 9} - \frac{4}{9 - A} < 0$
- and

$$\frac{A^2 - 5A}{A - 9} - \frac{4}{9 - A} = \frac{A^2 - 5A}{A - 9} + \frac{4}{A - 9} = \frac{(A - 1)(A - 4)}{A - 9}.$$

That is, this inequality can be written in the form of

$$\frac{(A - 1)(A - 4)}{A - 9} \leq 0.$$

By setting $(A - 1)(A - 4) = 0$ and $A - 9 = 0$, we consider the intervals divided by $A = 1, 4, 9$ as follows.

	$x \in (-\infty, 1)$	$x \in (1, 4)$	$x \in (4, 9)$	$x \in (9, \infty)$
$\frac{(A-1)(A-4)}{A-9}$	$\frac{(-)(-)}{(-)} = (-)$	$\frac{(+)(-)}{(-)} = (+)$	$\frac{(+)(+)}{(-)} = (-)$	$\frac{(+)(+)}{(+)} = (+)$

Note that we consider inequality with “ $\leq$ ” and so the points $A = 1, 4$ have to be included in the solution set, but the point $A = 9$ cannot be included since it would make denominator zero. Hence, from the table, the solution set for A is $(-\infty, 1] \cup [4, 9)$. ■

- (b) From (a), we can set
- $A = x^2$
- to obtain the inequality in (b). That is, using (a) gives

$$\frac{x^4 - 5x^2}{x^2 - 9} \leq \frac{4}{9 - x^2} \Leftrightarrow \frac{(x^2 - 1)(x^2 - 4)}{x^2 - 9} \leq 0$$

That is,

$$\frac{(x^2 - 1)(x^2 - 4)}{x^2 - 9} = \frac{(x + 1)(x - 1)(x + 2)(x - 2)}{(x + 3)(x - 3)} \leq 0$$

By setting $(x + 1)(x - 1)(x + 2)(x - 2) = 0$ and $(x + 3)(x - 3) = 0$, we consider the intervals divided by $A = -3, -2, -1, 1, 2, 3$ as follows.

	$x \in (-\infty, -3)$	$x \in (-3, -2)$	$x \in (-2, -1)$	$x \in (-1, 1)$	$x \in (1, 2)$	$x \in (2, 3)$	$x \in (3, \infty)$
$\frac{(x+2)(x+1)(x-1)(x-2)}{(x+3)(x-3)}$	$\frac{(-)(-)(-)(-)}{(-)(-)} = (+)$	$\frac{(-)(-)(-)(-)}{(+)(-)} = (-)$	$\frac{(+)(-)(-)(-)}{(+)(-)} = (+)$	$\frac{(+)(+)(-)(-)}{(+)(-)} = (-)$	$\frac{(+)(+)(+)(-)}{(+)(-)} = (+)$	$\frac{(+)(+)(+)(+)}{(+)(-)} = (-)$	$\frac{(+)(+)(+)(+)}{(+)(+)} = (+)$
	(+)	(-)	(+)	(-)	(+)	(-)	(+)

Note that we consider inequality with “ $\leq$ ” and so the points $x = -2, -1, 1, 2$ have to be included in the solution set, but the points $x = -3, 3$ cannot be included since they would make denominator zero. Hence, from the table, the solution set for x is $(-3, -2] \cup [-1, 1] \cup [2, 3)$. ■