# B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Section 046401)

Semester 1/2015

**Quiz 4 (a) - Answers**

Thursday November 12, 2015

Time: 11:10 – 11:25 hrs.

1. Consider the following production function

$$Q(K,L) = 32K + 18L - 2K^2 - 2KL - L^2.$$

Suppose that the output price is \$2 per unit, and the input prices for $K$ and $L$ are $r$ and $w$, respectively.

a. (6 *points*) Find the levels of $K^*$ and $L^*$ that maximize the firm's profit.

$$\max_{K,L} \pi(K,L) = 2(32K + 18L - 2K^2 - 2KL - L^2) - rK - wL$$

FONC:

$$\pi_K = 0 \rightarrow \pi_K = 2(32 - 4K - 2L) - r = 0 \quad \text{-- (1)}$$

$$\pi_L = 0 \rightarrow \pi_L = 2(18 - 2K - 2L) - w = 0 \quad \text{-- (2)}$$

$$\Rightarrow K^* = \frac{28-r+w}{4} \text{ and } L^* = \frac{8-2w+r}{4}$$

b. (2 *points*) Verify the answer in part (a) by using the second-order sufficient conditions.

SOSC:

$$H = \begin{bmatrix} -8 & -4 \\ -4 & -4 \end{bmatrix}$$

$\Rightarrow |H_1| = -8 < 0 \;\&\; |H_2| = |H| = 32 - 16 = 16 > 0$

$\Rightarrow \pi(K^*, L^*) = \pi\left(\frac{36-r+w}{4}, \frac{8-2w+r}{4}\right)$ is a maximum profit.

c. (2 *points*) Determine the impact of changes in $w$ on the optimal levels of inputs $K^*$ and $L^*$ (i.e. find $\frac{\partial K^*}{\partial w}$ and $\frac{\partial L^*}{\partial w}$), and evaluate the signs of both comparative statics derivatives.

$$\frac{\partial K^*}{\partial w} = \frac{\partial}{\partial w}\left(\frac{36 - r + w}{4}\right) = \frac{1}{4} > 0$$

$$\frac{\partial L^*}{\partial w} = \frac{\partial}{\partial w}\left(\frac{8 - 2w + r}{4}\right) = -\frac{1}{2} < 0$$