



FN 312

Investments



Challenge: How to invest optimally?

Short-term Treasury securities
(risk-free asset)

Stocks and other risky
assets: IBM, MSFT,
GE, INTC, GS, LEH

Long-term bonds



Outline

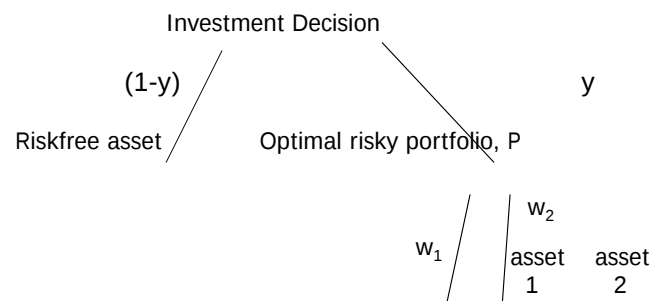
- The interest rate
- Return
- Variance risk
- Investors preference
- Risk-return trade off

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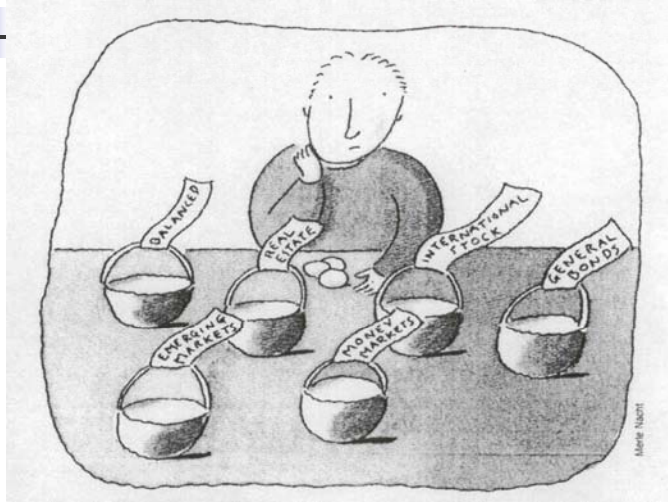
Investments

What is the optimal risky portfolio?



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What to do?



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Road map/Key ideas

- Simple diversification
- The effect of correlation on diversification
- Optimal diversification
- Limits to diversification

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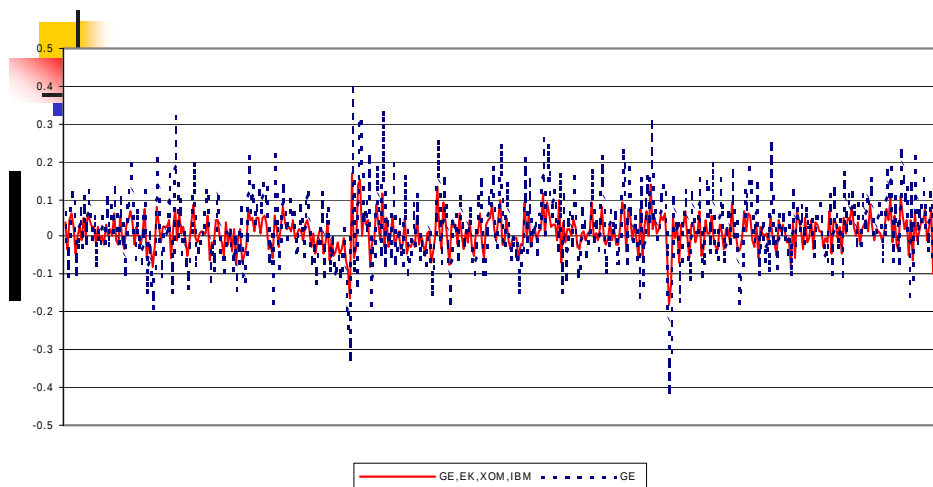
Over the period 1962 – 1998 (Annualized):

Portfolio	Average Rate of Return	Standard Deviation
MKT (Equal-weighted)	12.7%	15.1%
IBM	13.4%	22.0%
GE	16.1%	22.0%
EK	11.3%	21.8%
XOM	15.3%	16.8%

- The STD of the market portfolio is lower than most individual stocks

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Simple Diversification (1962-1998)



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Simple diversification (1962-1998)

Simple diversification: Equal-weighted portfolio

	IBM	GE	EK	XOM	MKT	IBM,GE	IBM,GE,EK	All 4 stocks
MEAN (per year)	0.134	0.161	0.113	0.153	0.127	0.147	0.136	0.140
STD (per year)	0.227	0.220	0.218	0.168	0.151	0.191	0.175	0.153

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Simple diversification

How diversification works

- Prices of different stocks do not move exactly together
- Variation of a stock price = market-wide variation + firm specific variation.
- Firm-specific variations of different stocks cancel each other out
- STD of a well-diversified portfolio $\leq$ STD of an individual stock that make up the portfolio

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Simple diversification



A Well-diversified portfolio should

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Simple diversification



A Well-diversified portfolio should

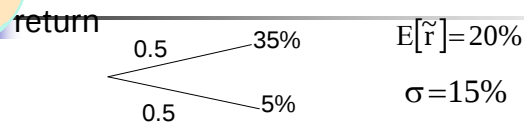
- Include large number of assets (>40)
- Include assets in different industry/market segment
- Include assets in different countries
- Consider the net current wealth including the present value of future income

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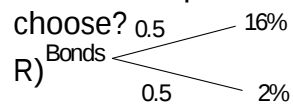


The effect of correlation on diversification

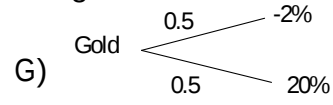
You have \$100,000 invested in the S&P, which will



Change composition: 70% is invest in the S&P and 30% in either corporate bonds or gold. Which one to choose?



$$E[\tilde{r}] = 9\% \quad \sigma = 0.07$$

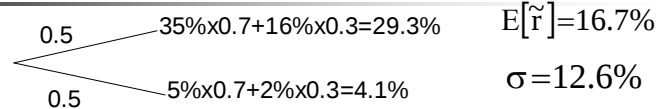


$$E[\tilde{r}] = 9\% \quad \sigma = 0.11$$

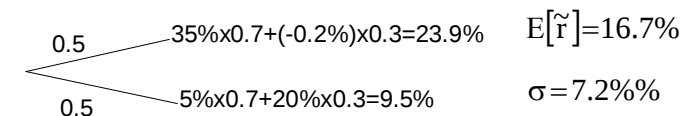
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The effect of correlation on diversification

1. 70% S&P + 30% Bonds



2. 70% S&P + 30% Gold



The correlation between two risky securities matters in obtaining a portfolio with lower risk

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The effect of correlation on diversification

Consider a portfolio P that invests w_1 in stock 1 and w_2 in stock 2. The correlation between the two stocks is ρ

$$E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

- Expected return
- Variance
- The bordered variance-covariance matrix

	w1	w2
w1	σ_1^2	$\rho\sigma_1\sigma_2$
w2	$\rho\sigma_1\sigma_2$	σ_2^2

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The effect of correlation on diversification

The bordered variance-covariance matrix for 3 assets

	w1	w2	w3
w1	σ_1^2	$\rho_{12}\sigma_1\sigma_2$	$\rho_{13}\sigma_1\sigma_3$
w2	$\rho_{12}\sigma_1\sigma_2$	σ_2^2	$\rho_{23}\sigma_2\sigma_3$
w3	$\rho_{13}\sigma_1\sigma_3$	$\rho_{23}\sigma_2\sigma_3$	σ_3^2

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The effect of correlation on diversification

Consider two risky assets: stock 1 and stock 2

- Stock 1: $E[\tilde{r}_1] = 8\% = \mu_1$, $\sigma[\tilde{r}_1] = \sigma_1 = 12\%$
- Stock 2: $E[\tilde{r}_2] = 13\% = \mu_2$, $\sigma[\tilde{r}_2] = \sigma_2 = 20\%$
- Riskfree rate is 0.5%

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2$$

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The effect of correlation on diversification

When $\rho=1$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2,$$

$$= (w_1 \sigma_1 + w_2 \sigma_2)^2$$

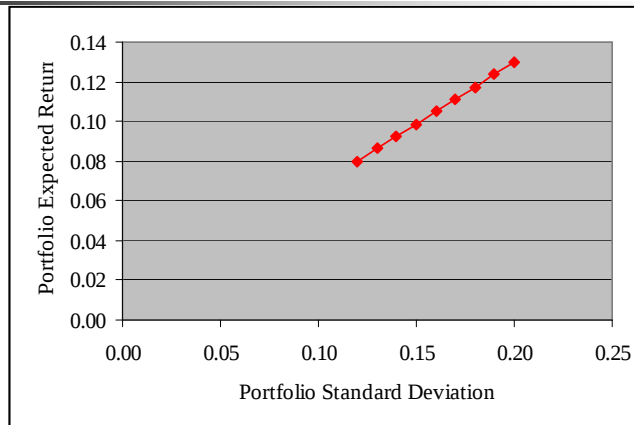
$$\sigma_p = w_1 \sigma_1 + w_2 \sigma_2$$

When the correlation equals 1 the STD of the portfolio equals the weighted average of the STD of the stocks that make up the portfolio. There is no diversification benefit.

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The effect of correlation on diversification

Case: $\rho=1$ there is no diversification benefit



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The effect of correlation on diversification

- When $\rho < 1$ there is always potential for diversification benefit

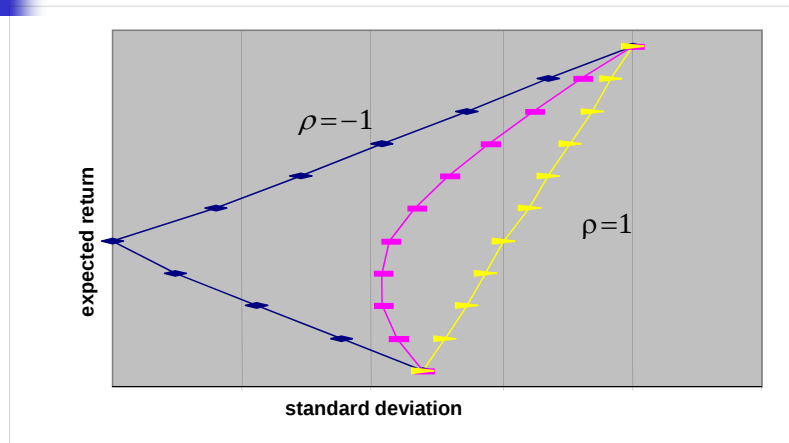
$$\sigma_P = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2} < w_1 \sigma_1 + w_2 \sigma_2$$

$\rho < 0$

- Assets with $\rho < 0$ are often referred to as hedged assets.

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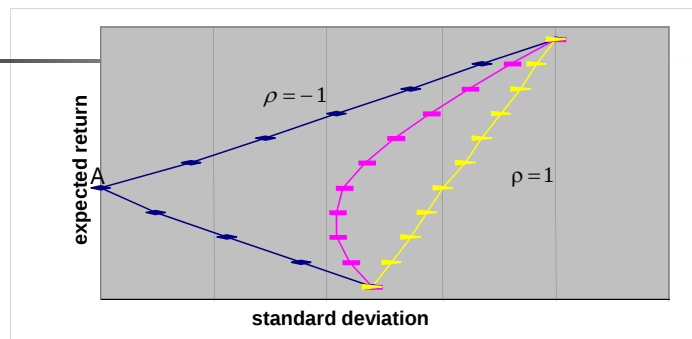
The effect of correlation on diversification



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Concept check



What should the expected return at point A be?

- G) Zero
- Y) Riskfree rate
- R) The return on the market portfolio

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The effect of correlation on diversification

Case: $\rho = -1$

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 - 2w_1 w_2 \sigma_1 \sigma_2$$

$$= (w_1 \sigma_1 - w_2 \sigma_2)^2$$

$$\sigma_p = w_1 \sigma_1 - w_2 \sigma_2$$

To find the point where volatility is zero

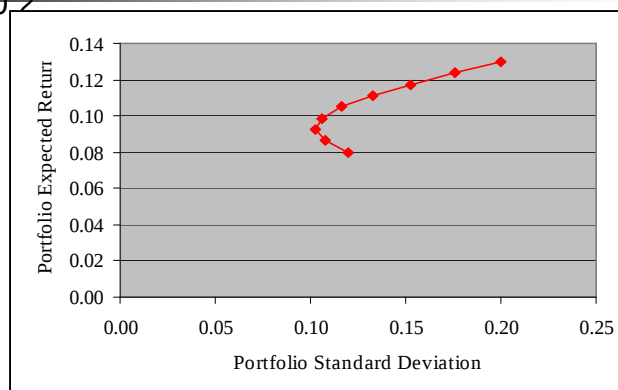
$$0 = w_1 \sigma_1 - (1 - w_1) \sigma_2$$

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

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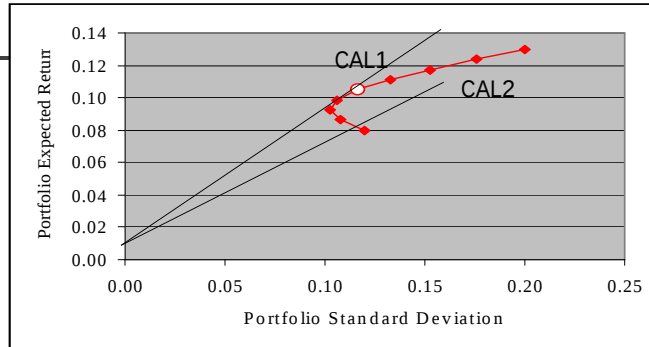
Optimal risky portfolio

Investment opportunity set of two risky assets when $\rho = -0.2$



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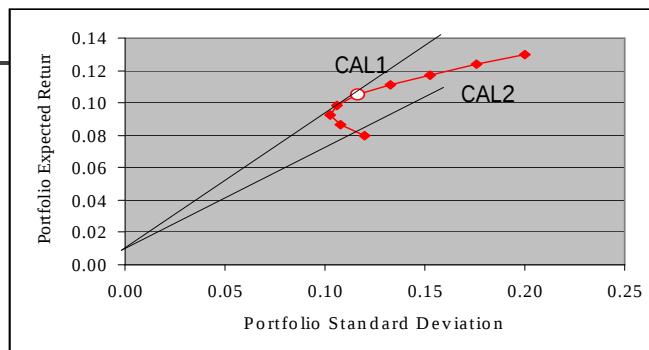
Optimal risky portfolio (visual)



- Adding the riskfree asset generates a number of CAL choices.
- Which CAL is better?

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Optimal risky portfolio (visual)



- Adding the riskfree asset generates a number of CAL choices.
- Which CAL is better?
- Choose the CAL with the highest slope (Sharpe ratio)

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Optimal risky portfolio (algebra)

Maximize $(E[\tilde{r}_p] - r_f) / \sigma_p$
 w_1, w_2

Subject to: $w_1 = (1 - w_2)$

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2$$

We obtain equation 8.7 in the book

$$w_1 = \frac{[E(r_1) - r_f] \sigma_2^2 - [E(r_2) - r_f] \text{Cov}(r_1, r_2)}{[E(r_1) - r_f] \sigma_2^2 + [E(r_2) - r_f] \sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f] \text{Cov}(r_1, r_2)}$$

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Optimal risky portfolio

Example: Investing \$100,000, riskfree=0.5%

■ Stock s 1 and 2: $E[\tilde{r}_1] = 8\%$, $\sigma_1 = 12\%$ $E[\tilde{r}_2] = 13\%$, $\sigma_2 = 20\%$

■ The correlation between the two stock returns is 0

2. Solve for P

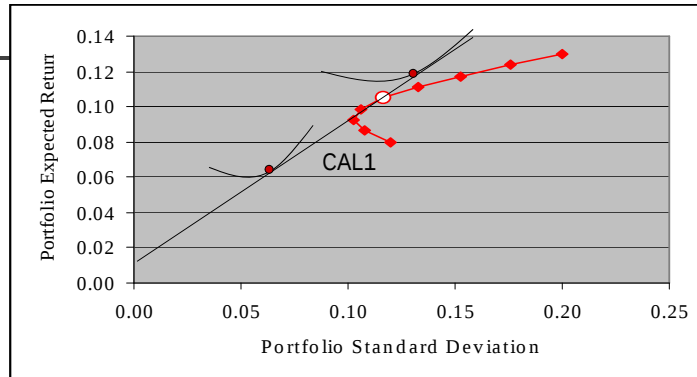
$$w_1 = \frac{[E(r_1) - r_f] \sigma_2^2 - [E(r_2) - r_f] \text{Cov}(r_1, r_2)}{[E(r_1) - r_f] \sigma_2^2 + [E(r_2) - r_f] \sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f] \text{Cov}(r_1, r_2)}$$

$$w_1 = \frac{[0.08 - 0.005] 0.2^2}{[0.08 - 0.005] 0.2^2 + [0.13 - 0.005] 0.12^2} = 0.625$$

$$w_2 = 0.375$$

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Optimal risky portfolio



- Allocate investment between riskfree asset and P using each investor's utility function

$$y^* = (E[\tilde{r}_p] - r_f) / (A \cdot \sigma_p^2)$$

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Optimal risky portfolio

Investing \$100,000, $A=8$, riskfree=0.5%

1. Stock s 1 and 2: $E[\tilde{r}_1] = 8\%$, $\sigma_1 = 12\%$ $E[\tilde{r}_2] = 13\%$, $\sigma_2 = 20\%$

- The correlation between the two stock returns is 0

2. We have $w_1 = 0.625$ and $w_2 = 0.375$

$$E[\tilde{r}_p] = 0.625 \times 0.08 + 0.375 \times 0.13 = 0.09875$$

$$\sigma_p = \sqrt{0.625^2 \cdot 0.12^2 + 0.375^2 \cdot 0.20^2 + 2 \times 0.625 \times 0.375 \times 0 \times 0.12 \times 0.20} = 0.106$$

3. $y^* = (0.09875 - 0.005) / (8 \times 0.106^2) = 1.04$

Borrow 4,000; Invest $0.625 \times 1.04 \times 100,000 = 65,000$ in stock 1; and $0.375 \times 1.04 \times 100,000 = 39,000$ in stock 2

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Concept check



Because diversification is so important, there is never a time when it is optimal to invest more than 90% of a portfolio in one specific asset. Is this true or false?

- G) True
- R) False

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Example: Investing \$100,000, riskfree=0.5%



Stock 1: $E[\tilde{r}_1] = 8\%$, $\sigma_1 = 12\%$

Stock 2: $E[\tilde{r}_2] = 13\%$, $\sigma_2 = 3\%$

- The correlation between the two stock returns is 0

2. Solve for P

$$w_1 = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]\text{Cov}(r_1, r_2)}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]\text{Cov}(r_1, r_2)}$$

$$w_1 = \frac{[0.08 - 0.005]0.03^2}{[0.08 - 0.005]0.03^2 + [0.13 - 0.005]0.12^2} = 0.036$$

$$w_2 = 0.964$$

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Limits to Diversification

Date 09/21/2001

Diversification Helps... but It Has Its Limits
Performance of various mutual-fund categories since last week's disasters and over longer periods. All returns through Wednesday.

Fund type	From Sept.10	Year-to-date	Past 3 years, annualized
Gold-oriented	+3.5%	+16.8%	+1.6%
Interm. U.S. govt. bond	+0.9	+6.9	+5.6
Money market	+0.03	+2.9	+4.9
Real-estate	-3.8	+1.6	+9.0
Balanced*	-1.2	-11.6	+3.1
International-stock	-5.9	-28.5	-1.1
International small-cap	-7	-28.4	+4.6
Diversified U.S.-stock	-7.9	-23.3	+4.1
Emerging markets	-9.4	-20.2	+3.4
Natural resources	-9.7	-20.1	+8.9

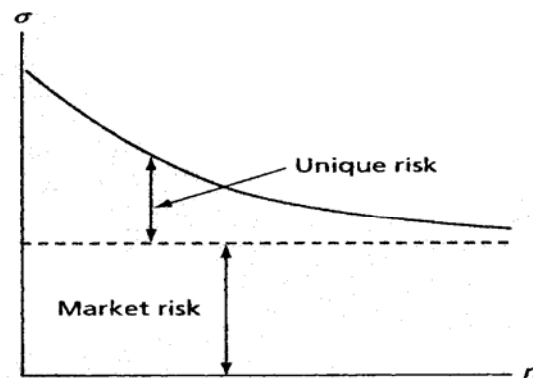
*Holds a mix of stocks and bonds. Source: Lipper

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Limits to Diversification

Total variation of a stock = market risk + unique risk

- Unique risk is diversified away, but market risk remains



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Summary

Simple diversification

- The effect of correlation on diversification
 - Portfolio of less than perfectly correlated assets always offer better risk-return opportunities than the individual component securities on their own
- Optimal diversification with two assets
 - Find the investment opportunity set. Pick the best CAL line.
- Limits to diversification
 - Firm specific risk is diversified away, but market risk remains