Members Gr. 1

6304640052

6304640136

6304640144

6304641035

1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

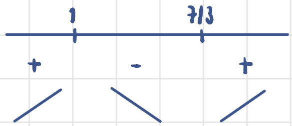
b. Define the domain set of X. Is the function concave for all over the domain?

a) F.O.C

$$\frac{dY}{dx} = 3x^2 - 10x + 7$$

set $\frac{dY}{dx} = 0$

$$3x^2 - 10x + 7 = 0$$
$$(3x - 7)(x - 1) = 0$$
$$x = 1, \frac{7}{3}$$



real number

when $f'(x)$ or $y' > 0$, it is implied that the function is increasing so, $(-\infty, 1) \cup (7/3, \infty)$ is where the function exhibits the property of an increasing function

b) S.O.C

$$\frac{d^2Y}{dx^2} = 6x - 10$$

the function will concave when $f''(x)$ or $y'' < 0$
thus, this function is not global concave since when x is more than $\frac{5}{3}$, it will be convex function $[f''(x) > 0]$

2. Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where $Q(L)$ is the output level, and L is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).

$$AP_L = \frac{Q(L)}{L} = 6L - L^2$$

$$MP_L = \frac{dQ(L)}{dL} = 12L - 3L^2$$

b. What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?

$$AP_L = 6L - L^2$$

F.O.C $\frac{dAP_L}{dL} = 0$

$$6 - 2L = 0$$

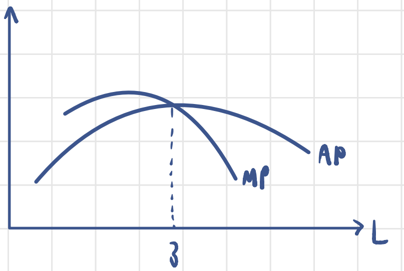
$L^* = 3$ units of labor

S.O.C $\frac{d^2AP_L}{dL^2} = -2$ always less than 0

it is a global concave function

so, $L^* = 3$ is the average output per labor maximizer.

c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.



Solve for L that MP_L curve cross the AP_L curve

$$MP_L = AP_L$$

$$12L - 3L^2 = 6L - L^2$$

$$0 = 2L^2 - 6L$$

$$0 = L(L-3)$$

$$L = 0, 3$$

from b) the maximum point of AP_L curve is $L=3$ which is where MP_L curve cross AP_L curve

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (*MRP*) function.

$$Q = 100 - 2P \qquad R = P(Q)\cdot Q$$

$$P = \frac{100-Q}{2} \qquad = 50Q - \frac{Q^2}{2}$$

$$\frac{dR}{dQ} = MRP = 50 - Q$$

3. Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 - 175Q + 10.$$

Consider the following questions.

a. Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?

$$R = P(Q)\cdot Q$$

$$= 40Q + 105 - \frac{3}{2}Q^3$$

F.O.C. $\frac{dR}{dQ} = 40 - \frac{9}{2}Q^2 = 0$

$$9Q^2 - 80 = 0$$

$$(3Q - \sqrt{80})(3Q + \sqrt{80}) = 0$$

impossible

$$Q = \frac{-4\sqrt{5}}{3}, \frac{4\sqrt{5}}{3} \quad ; \quad Q^* \approx 3$$

S.O.C $\frac{d^2R}{dQ^2} = -9Q$ at $Q = 3$ $\quad R''(3) = -27 < 0$

concave

so, $Q^* = 3$ is revenue-max solution #

$$\frac{dP}{dQ} = -105Q^{-2} - 3Q \quad ; \quad Q = 3 \Rightarrow \frac{dP}{dQ} = \frac{-62}{3}$$

$$\frac{P(Q)}{Q} = 40Q^{-1} + 105Q^{-2} - \frac{3}{2}Q \quad ; \quad Q = 3 \Rightarrow \frac{P(Q)}{Q} = \frac{40}{3} + \frac{35}{3} - \frac{9}{2} = \frac{41}{2}$$

$$\varepsilon^d = \frac{dQ}{dP} \times \frac{P}{Q} = -\frac{3}{62} \times \frac{41}{2} = \frac{-123}{124} \quad \#$$

elasticity of demand at $Q=3$

b. Construct the profit function.

$$\pi = P \cdot Q - C(Q)$$
$$= \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right)Q - (6Q^3 - 81Q^2 - 175Q + 10)$$
$$= 40Q + 105 - \frac{3}{2}Q^3 - 6Q^3 + 81Q^2 + 175Q - 10$$
$$\pi = -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 95 \quad \#$$

c. Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.

$$\pi = -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 95$$

F.O.C $\quad \frac{d\pi}{dQ} = 0$

$$-\frac{45}{2}Q^2 + 162Q + 215 = 0$$
$$45Q^2 - 324Q - 430 = 0$$
$$Q = \frac{324 \pm \sqrt{(-324)^2 - 4(45)(-430)}}{2(45)}$$
$$Q = \frac{324 \pm 427.06}{90} = 8.345, -1.145$$

impossible

S.O.C $\quad \pi'' = -45Q + 162$

$\pi''(Q = 8.345) = -45(8.345) + 162 = -213.525$ which is less than 0

it is concave at $Q = 8.345$, then $Q = 8.345$ is profit-max solution #

d. Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

lump-sum tax increases fixed cost of monopolist making average cost increases.
However, its price, quantity, and marginal cost remain the same. so, the amount of profit decreases.

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?

$\pi = P \cdot Q - STC$

$\pi = P \cdot q - 0.5q^2 - 10q - 5$

F.O.C. $\frac{d\pi}{dq} = 0$

$P - q - 10 = 0$

$q^* = P - 10$

S.O.C. $\frac{d^2\pi}{dq^2} = -1 < 0$

concave

then q^* is profit-max

$STC = 0.5q^2 + 10q + 5$

$TVC = 0.5q^2 + 10q$; $TFC = 5$

$AVC = 0.5q + 10$

$\min(AVC)$; $q = 0 \Rightarrow \min(AVC) = 10$

$P > \min(AVC)$

$P > 10$

$q^s = q^* = \begin{cases} 0 & ; P \leq 10 \\ P - 10 & ; P > 10 \end{cases}$ for each

$q^s_{market} = \begin{cases} 0 & ; P \leq 10 \\ 100P - 1000 & ; P > 10 \end{cases}$

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

Market demand = Market supply

$Q^d = \begin{cases} 0 & ; P \geq 22 \\ 1100 - 50P & ; P < 22 \end{cases}$ 1 2 $Q^s = \begin{cases} 0 & ; P \geq 10 \\ 100P - 1000 & ; P < 10 \end{cases}$

1) $1100 - 50P = 0$

$P = 22$ not < 22

2) $1100 - 50P = 100P - 1000$

$2100 = 150P$

$P^* = 14$

$Q^* = 400 \Rightarrow Q^*$ for each firm $= \frac{400}{100} = 4$

profit for each firm $\pi = (14)\left(\frac{400}{100}\right) - (0.5)(4^2) - 10(4) - 5$

$= 56 - 8 - 40 - 5$

$\pi = 3$ € #

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

$$P^d - t = P^s$$

$$Q^d = \begin{cases} 0 & ; P \geq 22 \\ 1100 - 50P & ; P < 22 \end{cases} \overset{\text{each}}{\Rightarrow} Q^d = 11 - 0.5P^d \Rightarrow P^d = 22 - 2Q^d$$

$$Q^s = \begin{cases} 0 & ; P \geq 10 \\ 100P - 1000 & ; P < 10 \end{cases} \overset{\text{each}}{\Rightarrow} Q^s = P^s - 10 \Rightarrow P^s = Q^s + 10$$

$$P^d - t = P^s$$

$$22 - 2Q - 3 = Q + 10$$

$$9 = 3Q$$

$Q^* = 3$ for each firm $\Rightarrow$ for market $Q^* = 300$

$$P^d = 16$$

$$P^s = 13$$

or

$$Q^d = \begin{cases} 0 & ; P \geq 22 \\ 1100 - 50P & ; P < 22 \end{cases} \Rightarrow P^d = \frac{1100 - Q^d}{50}$$

$$Q^s = \begin{cases} 0 & ; P \geq 10 \\ 100P - 1000 & ; P < 10 \end{cases} \Rightarrow P^s = \frac{1000 + Q^s}{100}$$

$$P^d - t = P^s$$

$$\frac{1100 - Q^d}{50} - 3 = \frac{1000 + Q^s}{100}$$

$$2200 - 2Q - 300 = 1000 + Q$$

$$900 = 3Q$$

$$Q^* = 300$$

$$P^d = \frac{1100 - 300}{50} = 16$$

$$P^s = \frac{1000 + 300}{100} = 13$$

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?

	before	after
Q^*	400	300
P^d	14	16
P^S	14	13

tax revenue = $3 \times 300 = 900$

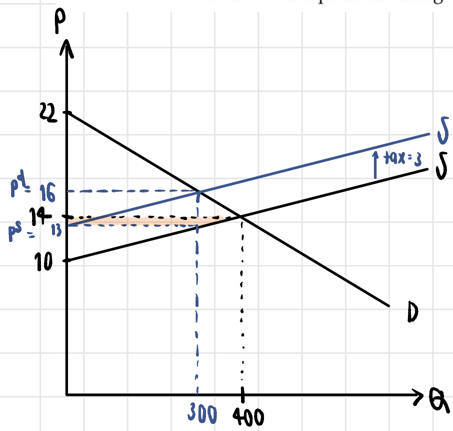
burden on owners $(16-14)(300) = 600$ ⇒ 67 %

burden on firms $(14-13)(300) = 300$ ⇒ 33 %

burden per pool for owners = 2 €

burden per pool for firms = 1 €

e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.



Loss in producer surplus = $\frac{1}{2}(13-14)(400+300) = -350$

- before tax

from b) profit for each firm = 3

profits for industry = 300

- after tax

profit for each firm

$= (13 \times 3) - (0.5)(3^2) - (10)(3) - 5 = -0.5$

profits for industry = -50

change in total short run profits in this industry = $-50 - 300 = -350$

Thus, the loss of producer surplus equal to change in total short run profits in this industry which is -350 #