

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\begin{aligned} \text{a. } \sum_{i=1}^5 (a + bx_i) &= 5a + b \sum_{i=1}^5 x_i \\ &= 5a + b (x_1 + x_2 + x_3 + x_4 + x_5) \end{aligned}$$

$$\text{b. } \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$\text{c. } \sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$$

$$\text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 [2x + 2 + 3] = \sum_{x=1}^2 2x + 5 = 2(1) + 2(2) + 5 = 11$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$$\begin{aligned} f(x) = P(X=x) &= 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1 \\ 8b &= 1 \\ b &= \frac{1}{8} \end{aligned}$$

b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= 1 - P(X > 2) \\ &= 1 - P(X=3) - P(X=4) \\ &= 1 - (0.5)(0.125) - (0.25)(0.125) = 0.90625 \end{aligned}$$

c. Find the answer for $P(-2 \leq X \leq 3)$

$$\begin{aligned} P(-2 \leq X \leq 3) &= 1 - P(X=4) \\ &= 1 - (0.25)(0.125) \\ &= 0.96875 \end{aligned}$$

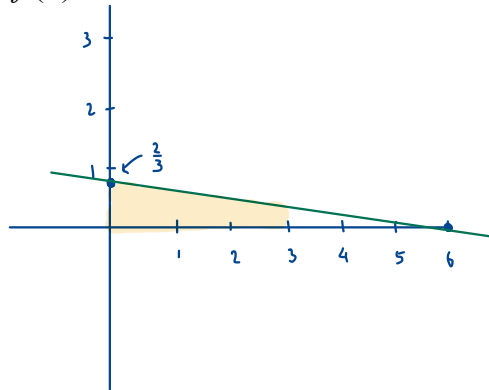
d. Find the answer for $P(X \geq 1)$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X=0) - P(X=-1) - P(X=-2) \\ &= 1 - (2.25)(0.125) - (1)(0.125) - (0.5)(0.125) \\ &= 0.53125 \end{aligned}$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 f(x) dx \\ &= \left. -\frac{1}{18}x^2 + \frac{6}{9}x \right|_1^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9} \right] = \frac{-9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9} = \frac{-8}{18} + \frac{12}{9} = \frac{16}{18} \end{aligned}$$

c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(X \geq 2) &= \int_2^3 f(x) dx \\ &= \left. -\frac{1}{18}x^2 + \frac{6}{9}x \right|_2^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9} \right] = \frac{-9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9} = \frac{7}{18} \end{aligned}$$

d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left. -\frac{x^3}{27} + \frac{6x^2}{18} \right|_0^3 \\ &= -\frac{(3)^3}{27} + \frac{6(3)^2}{18} \\ &= -\frac{27}{27} + \frac{6 \cdot 9}{18} \\ &= -1 + 3 \\ &= 2 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

a. Construct the joint probability distribution function (PDF) table of X and Y

X/Y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

b. Find the marginal probability distribution function (PDF) of X

the marginal probability of x $P(X=x)$ is $\frac{1}{6}$

c. Find the marginal probability distribution function (PDF) of Y

the PDF, $P(Y=y)$ is $\frac{1}{2}$

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1 $P(X|Y=1)$

X	1	2	3	4	5	6
$P(X=x Y=1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x_i P(X=x_i|Y=1) = \frac{\sum x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1}{0.5} \sum x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5} \left[(1 \cdot \frac{1}{12}) + (2 \cdot \frac{1}{12}) + (3 \cdot \frac{1}{12}) + (4 \cdot \frac{1}{12}) + (5 \cdot \frac{1}{12}) + (6 \cdot \frac{1}{12}) \right] = \frac{7}{2}$$

f. Find the variance of X given Y is equal to 1

$$Var(X|Y=1) = \sum (x - E(X|Y=1))^2 \cdot P(X|Y=1)$$

$$= \left[\left((1 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((2 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((3 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((4 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) \right.$$

$$\left. + \left((5 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((6 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) \right]$$

$$= \frac{10}{3}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

\bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3) \\ &= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3)] \\ &= \frac{1}{3} [\mu_x + \mu_x + \mu_x] \\ &= \frac{1}{3} \cdot 3\mu_x = \mu_x \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3) \\ &= \frac{1}{N^2} \left[\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) \right. \\ &\quad \left. + 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_1, X_3) \right. \\ &\quad \left. + 2\text{Cov}(X_2, X_3) \right] \\ &= \frac{1}{3^2} \left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 \right] \\ &= \frac{1}{3^2} \cdot \frac{9}{4}\sigma_x^2 = \frac{\sigma_x^2}{4} \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . \bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3) + E(X_4)] \\ &= \frac{1}{4} [\mu_x + \mu_x + \mu_x + \mu_x] \\ &= \frac{1}{4} \cdot 4\mu_x = \mu_x \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N^2} \left[\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \text{Var}(X_4) \right] \\ &= \frac{1}{4^2} \left[\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2 \right] \\ &= \frac{1}{16} \cdot 4\sigma_x^2 = \frac{\sigma_x^2}{4} = 0.25\sigma_x^2 \end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that \tilde{X} is an unbiased estimator of μ

$$\tilde{X} = \frac{1}{4} (0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$E(\tilde{X}) = E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4} E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4} [E(0.5X_1) + E(X_2) + E(0.5X_3) + E(2X_4)]$$

$$= \frac{1}{4} [0.5E(X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)]$$

$$= \frac{1}{4} (0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

\tilde{X} is unbiased estimator of μ

$$\text{Var}(\tilde{X}) = \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4^2} \text{Var}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4^2} [\text{Var}(0.5X_1) + \text{Var}(X_2) + \text{Var}(0.5X_3) + \text{Var}(2X_4)]$$

$$= \frac{1}{4^2} [0.25 \text{Var}(X_1) + \text{Var}(X_2) + 0.25 \text{Var}(X_3) + 4 \text{Var}(X_4)]$$

$$= \frac{1}{4^2} [0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2]$$

$$= \frac{5.5\sigma_x^2}{16}$$

$$= 0.34\sigma_x^2$$

- c. Between \bar{X} and \tilde{X} , which one is the better estimator for μ ? Why?

\bar{X} is a more efficient estimator of μ_x than

\tilde{X} because it has smaller variance.