0.1) Given that $Z = \frac{x^3-y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial Z}{\partial x} = \frac{3x^2(x^2y^2) - 2xy^2(x^3-y^3)}{(x^2y^2)^2}$$

$$= \frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{x^4y^4}$$

$$= \frac{3x^3 - 2x^3 + 2y^3}{x^3y^2}$$

$$\frac{\partial Z}{\partial y} = \frac{-3y^2(x^2y^2) - 2x^2y(x^3-y^3)}{(x^2y^2)^2}$$

$$= \frac{-3x^2y^4 - 2x^5y + 2x^2y^4}{x^4y^4}$$

$$= \frac{-3y^3 - 2x^3 + 2y^3}{x^2y^3}$$

$$-Z = x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial x}$$

$$-Z = x\left(\frac{3x^3-2x^3+2y^3}{x^3y^2}\right) + y\left(\frac{-3y^3-2x^3+2y^3}{x^2y^3}\right)$$

$$-Z = \frac{3x^3-2x^3+2y^3-3y^3-2x^3+2y^3}{x^2y^2}$$

$$-Z = \frac{x^3-y^3-2x^3+2y^3}{x^2y^2}$$

Given that $Z = \frac{x^3-y^3}{x^2y^2}$

$$-\left(\frac{x^3-y^3}{x^2y^2}\right) = \frac{-x^3+y^3}{x^2y^2}$$

$$\frac{-x^3+y^3}{x^2y^2} = \frac{-x^3+y^3}{x^2y^2} \ \#$$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

$$dZ = \frac{\partial Z}{\partial x} \cdot dx + \frac{\partial Z}{\partial y} \cdot dy$$

$$dZ = \left[\frac{1 \cdot (x+y) - 1 \cdot (x-y)}{(x+y)^2}\right] \cdot dx + \left[\frac{-(x+y) - 1 \cdot (x-y)}{(x+y)^2}\right] \cdot dy$$

$$dZ = \left[\frac{x+y-x+y}{(x+y)^2}\right] \cdot dx + \left[\frac{-x-y-x+y}{(x+y)^2}\right] \cdot dy$$

$$dZ = \frac{2y}{(x+y)^2} \cdot dx - \frac{2x}{(x+y)^2} \cdot dy$$

Given that $dx = 2$ and $dy = -2$;

$$dz\Big|_{x=1, y=1} = \frac{2(1)}{(1+1)^2} \cdot (2) - \frac{2(1)}{(1+1)^2} \cdot (-2)$$

$$= 1 + 1$$

$$= 2 \#$$

0.3) If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

0.3a) $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$

$= (4xy+3y)(2r) + (2x^2+3x+2y)(-4)$ #

$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$

$= (4xy+3y)(2r+2s) + (2x^2+3x+2y)(2)$ #

0.3b) When r=1 and s=0 ;

$x|_{r=1,s=0} = (1)^2 + 2(1)(0) = 1$

$y|_{r=1,s=0} = 2(1) - 4(0) = 2$

$\left.\frac{\partial z}{\partial s}\right|_{r=1,s=0} = (4(1)(2) + 3(2))(2(1)) + (2(1)^2 + 3(1) + 2(2))(-4)$

$= (8+6)(2) + (2+3+4)(-4)$

$= 28 - 36$

$= -8$ #

$\left.\frac{\partial z}{\partial r}\right|_{r=1,s=0} = (4(1)(2) + 3(2))(2(1) + 2(0)) + (2(1)^2 + 3(1) + 2(2))(2)$

$= (8+6)(2) + (2+3+4)(2)$

$= 28 + 18$

$= 46$ #

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

$$2x^2 + 3y^2 + 2z^2 = 16$$

$$f(x,y,z) = 2x^2 + 3y^2 + 2z^2 - 16$$

$$\frac{\partial 0}{\partial y} = \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial y} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y}$$

$$0 = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

$$\frac{\partial f}{\partial y} = 6y \; ; \; \frac{\partial f}{\partial z} = 4z$$

$$\frac{\partial z}{\partial y} = -\left(\frac{6y}{4z}\right) = -\frac{3y}{2z}$$

$$\left.\frac{\partial z}{\partial y}\right|_{x=1, y=2, z=-1} = -\left(\frac{3(2)}{2(-1)}\right) = 3 \; \#$$

0.5) Given that $ln\,(x+y+z)+xyz=ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

$$f(x,y,z)=\ln(x+y+z)+xyz-ze^{x+y+z}$$

$$\frac{\partial 0}{\partial x}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial x}+\frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$0=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}$$

$$\frac{\partial f}{\partial x}=\left(\frac{1}{x+y+z}\right)+yz-(ze^{x+y+z})(1)$$

$$\frac{\partial f}{\partial z}=\left(\frac{1}{x+y+z}\right)+xy-(ze^{x+y+z})(1)$$

$$\frac{\partial z}{\partial x}=-\left[\frac{\left(\frac{1}{x+y+z}\right)+yz-ze^{x+y+z}}{\left(\frac{1}{x+y+z}\right)+xy-ze^{x+y+z}}\right]$$

$$\left.\frac{\partial z}{\partial x}\right|_{0,1,0}=-\left[\frac{\left(\frac{1}{0+1+0}\right)+0-0}{\left(\frac{1}{0+1+0}\right)+0-0}\right]$$

$$=-\left(\frac{1}{1}\right)$$

$$=-1\#$$

Question 2 Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

According to the properties of cobb-doulass function, the production function exhibits a decreasing return to scale when $n+n<1$.

$2n<1$

$n<0.5$ #

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.

$Q = AK^n + AL^n$

$\frac{\partial Q}{\partial L} = \frac{AnL^n}{L}$ $\quad \frac{\partial^2 Q}{\partial L^2} = (n-1)AnL^{n-2}$; $n<1$

$\therefore$ As one more unit of labor is added, the marginal product of labor falls by $(n-1)AnL^{n-2}$

$\frac{dQ}{dK} = \frac{AnK^n}{K} \longrightarrow \frac{\partial^2 Q}{\partial K^2} = (n-1)AnK^{n-2}$; $n<1$

$\therefore$ As one more unit of capital is added, the marginal product of capital falls by $(n-1)AnK^{n-2}$

2.3) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

$Q = AK^n + AL^n$

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}} = \frac{AnL^{n-1}}{AnK^{n-1}} = \frac{L^{n-1}}{K^{n-1}} \ \#$$

2.4) how that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

$$\frac{\partial MRTS}{\partial L} = \frac{1}{K^{n-1}}(n-1)L^{n-2} = \frac{(n-1)L^{n-2}}{K^{n-1}} \ ; n<1$$

The MRTS reflects that the producer can replace the capital with labor at the rate of $\frac{(n-1)L^{n-2}}{K^{n-1}}$ though as more labors are employed, the MP_L decreases, resulting in a decrease of MRTS as it's derived from $\frac{MP_L}{MP_K}$.

Suppose that $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

2.5) Show that Q is increasing over time.

$$Q = A[K^n + L^n]$$
$$= A[(\tfrac{1}{2}t^2+2t+3)^n + (e^t+3)^n]$$
$$= A(\tfrac{1}{2}t^2+2t+3)^n + A(e^t+3)^n$$

$$\frac{\partial Q}{\partial t} = An(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + An(e^t+3)^{n-1}(e^t)$$
$$= An\left[(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + (e^t+3)^{n-1}(e^t)\right] \#$$

2.6) Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.

$$\frac{\frac{dQ}{Q}}{dt} = \frac{dQ}{Q} \cdot \frac{1}{dt}$$

$$Q = A[K^n + L^n]$$

$$= \frac{dQ}{dt \cdot Q}$$

$$= \frac{dQ}{dt} \cdot \frac{1}{Q}$$ From 2.5), we know $\frac{\partial Q}{\partial t}$;

$$= \left[An(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + An(e^t+3)^{n-1}(e^t)\right]\frac{1}{Q}$$

$$= \left[An\left[(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + (e^t+3)^{n-1}(e^t)\right]\right]\frac{1}{Q}$$

$$= \frac{An\left[(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + (e^t+3)^{n-1}(e^t)\right]}{A[K^n+L^n]}$$

$$= \frac{n\left[(\tfrac{1}{2}t^2+2t+3)^{n-1}(t+2) + (e^t+3)^{n-1}(e^t)\right]}{[K^n+L^n]}$$

$$= \frac{n\left[(\tfrac{1}{2}(0)^2+2(0)+3)^{n-1}(0+2) + (e^0+3)^{n-1}(e^0)\right]}{K^n+L^n}$$

$$= \frac{n[(0+0+3)^{n-1}(2) + (1+3)^{n-1}(1)]}{K^n+L^n}$$

$$= \frac{2n(3)^{n-1} + 4^{n-1}}{K^n+L^n}$$

$$= \left[\frac{2n(3)^n}{3} + \frac{4^n}{4}\right]\frac{1}{K^n+L^n}$$

$$= \left[\frac{8n(3)^n + 3(4)^n}{12}\right]\frac{1}{K^n+L^n}$$

When $t=0$;

$K(0)=3$

$L(0)=4$

$$= \left[\frac{8n(3)^n + 3(4)^n}{12}\right]\frac{1}{3^n+4^n}$$

$$= \frac{8n(3)^n + 3(4)^n}{12(3^n+4^n)} \#$$

Question 4: Profit function : Revenue function - Cost function

Revenue function = $P \times Q$, Cost function = $w \times L$

$\therefore$ Profit function = $(P \times Q) - (w \times L)$ = $PL^{1/2} - WL$; when $Q = L^{1/2}$

FOC : Max Profit ; MR = MC

Find MR : $R = P \times Q$, so MR = P

MC : $C = WL = WQ^2$, so MC = 2WQ

$\therefore \quad P = 2WQ$ or $P = 2WL^{1/2}$

$$L^{1/2} = \frac{P}{2W}$$

$$L^* = \left(\frac{P}{2W}\right)^2 \#$$

$$\therefore \quad L^* = \frac{1}{4} \cdot P^2 W^{-2} \quad ; L^* > 0$$

Find $\frac{\partial L^*}{\partial W} = \frac{1}{4} P^2 \cdot (-2) \cdot W^{-3} = \frac{-P^2}{2W^3} \#$

$$\frac{\partial L^*}{\partial P} = \frac{1}{4} W^{-2} \cdot 2 \cdot P = \frac{P}{2W^2} \#$$

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