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EE431 Economics of Financial Markets and Institutions
Problem Set 5: Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT)

1. Assume that all assumptions of CAPM hold.

Assets	Standard Deviation (%)
Security 1	4
Security 2	7
Security 3	9
Security 4	10
Security 5	14

Expected return on the market portfolio ($E(R_m)$) is equal to 0.08 and standard deviation of the market portfolio is equal to 0.10. The risk-free rate (R_f) is equal to 0.02.

(a) Write down CML equation.

ANSWER.

$$\begin{aligned} E(R_p) &= R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right)\sigma_p \\ &= 0.02 + \left(\frac{0.08 - 0.02}{0.10}\right)\sigma_p \\ &= 0.02 + 0.6\sigma_p \end{aligned}$$

CML equation : $E(R_p) = 0.02 + 0.6\,\sigma_p$.

(b) Suppose an investor wants 6% expected rate of return, how should the investor allocate his or her fund between the market portfolio and risk fee asset? What is the standard deviation of the portfolio?

ANSWER.

$$\begin{aligned} E(R_p) &= (1-a)R_f + aE(R_m) \\ 0.06 &= (1-a)(0.02) + a(0.08) \\ 0.06 &= 0.02 - 0.02a + 0.08a \\ 0.04 &= 0.06a \\ a &= \frac{0.04}{0.06} \\ &= \frac{2}{3} \\ \sigma_p &= a\sigma_m \\ &= \frac{2}{3} \times 0.10 = 0.0\dot{6} \end{aligned}$$

(c) "According to CAPM, the price of security 5 must be higher than the price of security 3 because security 5 has a higher risk than security 3." Is the statement true? Is the given information sufficient? If not, specify what piece of information you need.

ANSWER.

The statement is **not (necessary) true**.

According to CAPM, all investors hold an efficiently diversified portfolio comprising the market portfolio and the risk-free asset. People consider how individual asset contributes in their asset portfolio. Therefore, **variance is not a good measure of the risk of an individual asset or any portfolio apart from efficiently diversified portfolio.** Some risk is diversifiable, called unsystematic risk. Investors do not need compensation for unsystematic risk. Some risk is non-diversificable, called systematic risk. Investors need compensation for systematic risk.

"However, no level of diversification can reduce or get rid of the second component of risk. This is known as market or systematic risk and refers to the intrinsic risk of investing in the stock market given thath companies are subject to greater volatility than many other financial investments such as government bonds. There are external business environmental factors, such as worldwide recessions and economic boom, which affect all businesses, but they do not affect all businesses to the same extent."

According to CAPM, a good measure of the risk of any asset or any portfolio is the covariance risk. Covariance risk is the measure of any asset's systematic risk. Covariance risk can be measure by using covariance between return of the asset and the market portfolio. $COV(R_i, R_m)$ can take any values. The other measure of covariance risk is an asset's beta, $\beta_i = \dfrac{COV(R_i, R_m)}{\sigma_m^2}$. Measuring systematic risk this way, the size of the market portfolio's systematic risk is normalized to one.

"The relative sensitivity of a particular company's return to changes in the returns of the total stock market is called "*beta*" of the company. If the company is twice sensitive as the stock market, it is said to have a beta of 2; i.e. when overall stock market returns increase by 5 %, the returns on this share rise by 10%. Simiarly, a less sensitive company may have a beta of 0.75 which means that if the returns on stock market falls by 10%, this company returns will only fall by 7.5%."

The information on standard deviation (and hence variance) of security 3 and security 5 are given. Security 5 has larger variance than security 3. However, it is not necessary that security 5 will have higher beta (β) than security 3. **Higher risk implies higher required rate of return and lower price**. Therefore, **it is not necessary that security 5 will have a lower price than security 3**. **The given information is insufficient. We need more information on covariance between the return of security 5 and the return on market $[COV(R_5, R_m)]$ and on covariance between the return on security 3 and the return on the market $[COV(R_3, R_m)]$. Then, we can calculate β for each security and can compare the level of systematic risk between the two assets. We can calculte the required rate of return of both security by using the CAPM, $E(R_i) = R_f + [E(R_m) - R_f]\,\beta_i$.**

2. Assume that stock returns are generated by a one-factor model, the returns on two stocks, A and B are given by the following representations:

   E$R_A = 0.09 + 2F_1$

   $ER_B = 0.05 + F_1$

   Assuming that the conditions of Arbitrage Pricing Theory (APT) hold.

   (a) Write down the general formula for one-factor model.

   ANSWER.

   $w_a =$ weight put on asset A

   $w_b =$ weight put on asset B

   Find a portfolio which has zero risk.

$$\begin{aligned} E(R_p) &= w_a R_a + w_b R_b \\ &= w_a(0.09 + 2F_1) + w_b(0.05 + F_1) \\ &= (0.09w_a + 0.05w_b) + (2w_a + w_b)F_1 \end{aligned}$$

   A risk free portfolio will have $\beta_1 = 0$.

$$\begin{aligned} 2w_a + w_b &= 0 && --- \quad (1) \\ w_a + w_b &= 1 && --- \quad (2) \end{aligned}$$

$$\begin{aligned} 2w_a + 1 - w_a &= 0 \\ w_a &= -1 \\ w_b &= 2 \\ R_0 &= 0.09(-1) + 2(0.05) \\ &= -0.09 + 0.10 \\ &= 0.01 \end{aligned}$$

   The general form is

   $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i + ... + (\lambda_k - R_f)\beta_k^i$;

   $\lambda_j =$ the expected rate of return on portfolio wit

   h unit sensitivity to the j the factor and zero sensitivity to all other factors.

   Here, we have one-factor model. R_f is return on portfolio which has $\beta_1 = 0$ and λ_1 is return on portfolio which has $\beta_1 = 1$. $R_f = R_0$ and $R_1 = R_B$.

   Then, the general formula is as follows.

$$\begin{aligned} E(R_i) &= 0.01 + \beta_i(0.05 + F_1 - 0.01) \\ &= (0.01 + 0.04\beta_i) + \beta_i F_1 \end{aligned}$$

   (b) Suppose $ER_c = 0.01$. How to make an arbitrage profit?

   ANSWER. Asset C has no risk and its return is equal to the risk-free portfolio. Therefore, there is no arbitrage opportunity.

3. Assume that stock returns are generated by a two-factor model, the returns on two stocks, A and B are given by the following representations:

   E$R_A = 0.01$

   $ER_B = 0.06 + F_1$

   $ER_C = 0.05 + F_2$

   Assuming that the conditions of Arbitrage Pricing Theory (APT) hold.

   Write down the general formula for two-factor model.

   ANSWER. The general form is

   $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i + ... + (\lambda_k - R_f)\beta_k^i$;

   λ_j = the expected rate of return on portfolio with unit sensitivity to the j the factor and zero sensitivity to all other factors. *

   Here, we have one-factor model. R_f is return on portfolio which has $\beta_1 = 0$. λ_1 is return on portfolio which has $\beta_1^i = 1$ and $\beta_2^i = 0$. λ_2 is return on portfolio which has $\beta_1^i = 0$ and $\beta_2^i = 1$. $R_f = R_A$, $\lambda_1 = R_B$ and $\lambda_2 = R_C$.

   $$\begin{aligned} E(R_i) &= 0.01 + (0.06 + F_1 - 0.01)\beta_1^i + (0.05 + F_2 - 0.01)\beta_2^i \\ &= \left(0.01 + 0.05\beta_1^i + 0.04\beta_2^i\right) + \beta_1^i F_1 + \beta_2^i F_2 \end{aligned}$$

   * **Notice**.

   An asset which $\beta_1^i = 1$ and $\beta_2^i = 0$ will have return equal to λ_1. Substitute $\beta_1^i = 1$ and $\beta_2^i = 0$ in to the general equation.

   $$\begin{aligned} E(R_i) &= R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i \\ &= R_f + (\lambda_1 - R_f)(1) + (\lambda_2 - R_f)(0) \\ &= R_f + \lambda_1 - R_f \\ &= \lambda_1 \end{aligned}$$

   That is how we define λ_j.