Tumnurat Pucksanonthachai 6204641077

Assignment 1

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

Estimator of β_2 = $\hat{\beta}_2 = \frac{\sum(Xi - \overline{X})(Yi - \overline{Y})}{\sum(Xi - \bar{X})^2} = \frac{-174.20}{1098.8}$ = -0.1585

Estimator of β_1 = $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$ = 21.03 – (-0.1585)x(12.20) = 22.9637

The regression model = $\widehat{Yi}$ = 22.9637 + (-0.1585)$\hat{X}i$

Slope = X increases by 1, Y decrease by 0.1585 unit holding others constant

Intercept = Assumed X = 0, average or conditional mean of Y = 22.9637

b) Find r^2 and explain its meaning.

$r^2 = \frac{Explained\ Sum\ Squares}{Total\ Sum\ of\ Squares} = \frac{ESS}{TSS}$

$= 1 - \frac{Residual\ Sum\ of\ Squares}{Total\ Sum\ of\ Squares} = 1 - \frac{\sum(Yi - \hat{Y}i)^2}{\sum(Yi - \bar{Y})^2} = 1 - \frac{\sum \hat{U}i^2}{\sum(Yi - \bar{Y})^2} = 1 - \frac{873.14}{882.97} = 0.0111$

Explained Part, X can explain Y for 0.0111, 1.11%

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$E(\hat{Y}i|Xi=5) = \widehat{Yi}$ = 22.9637 + (-0.1585)x(5) = 22.1712

Average or conditional mean of Y = 22.1712 when X = 5

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

K = 2, 2 parameters estimated ($\hat{\beta}_1, \hat{\beta}_2$)

Estimator of Var(u_i) = $\hat{\sigma}^2 = \frac{\sum \hat{u}i^2}{n-k} = \frac{873.14}{30-2} = 31.1836\%^2$

Estimator of var ($\hat{\beta}_1$) = $\hat{\sigma}^2_{\hat{\beta}1} = \frac{\sum Xi^2}{n\sum xi^2}\sigma^2 = \frac{\sum Xi^2}{n\sum(Xi-\bar{X})^2}\sigma^2 = \frac{(5{,}564)(31.1836)}{30(1098.8)} = 5.2635\%^2$

Estimator of var ($\hat{\beta}_2$) = $\hat{\sigma}^2_{\hat{\beta}2} = \frac{\sigma^2}{\sum xi^2} = \frac{31.1836}{1098.8} = 0.0284\%^2$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

Level of significance = $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

Step 1: Hypothesis Testing

H_0: $\beta_2 = 0$ -> Null Hypothesis

H_a: $\beta_2 \neq 0$ -> Alternative Hypothesis

Step 2: Calculate test statistics

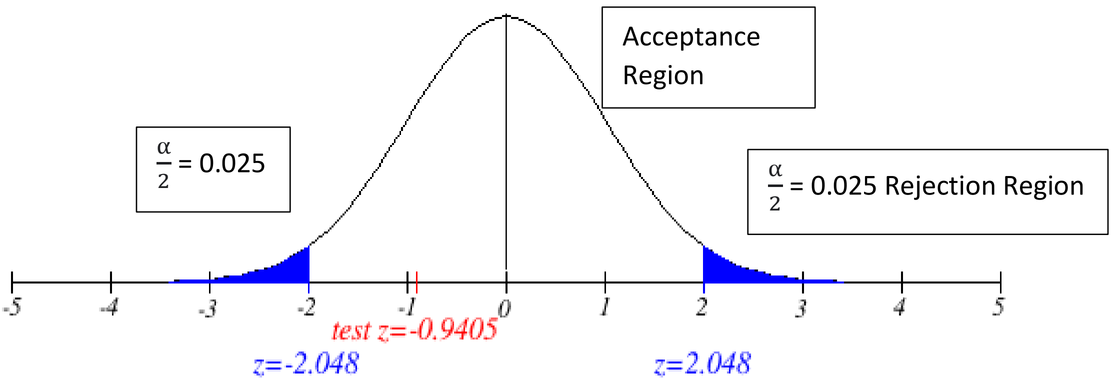
$t_{cal} = \frac{\widehat{\beta_2}-\beta_2}{\sigma_{\widehat{\beta 2}}} = \frac{-0.1585-0}{\sqrt{0.0284}} = -0.9405$

Step 3: State decision rule

n-k = 30-2 = 28, $\alpha = 0.05$

The Lower bound: $t_{\frac{\alpha}{2}} = -2.048$

The Upper bound: $t_{\frac{\alpha}{2}} = 2.048$



Step 4: $t_{cal} = -0.9405$ is in acceptance region. We don’t reject the null hypothesis at significant level of 95%. We can’t say for sure that β_2 is not 0, 95 out of 100 times.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

Level of significance = α = 0.01

Step 1: Hypothesis Testing

H_0: $\beta_2 \leq 0$ -> Null Hypothesis

H_a: $\beta_2 > 0$ -> Alternative Hypothesis

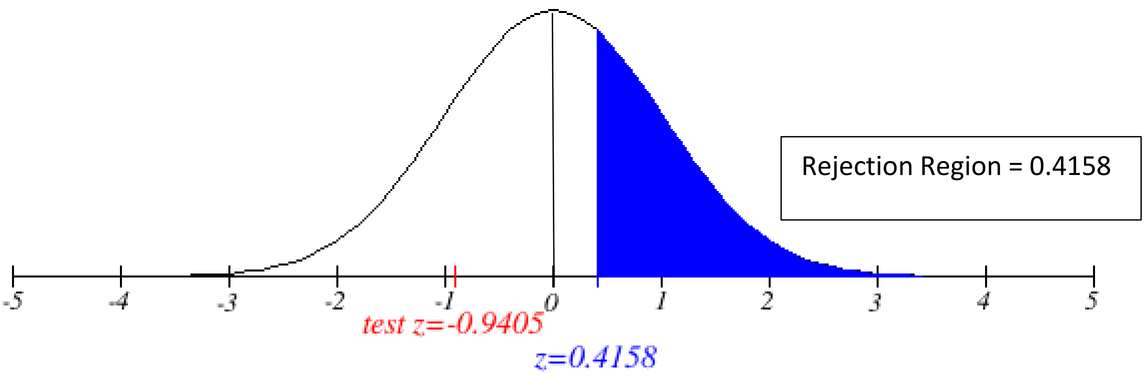
Step 2: Test Statistics

$t_{cal} = \frac{\widehat{\beta_2} - \beta_2}{\sigma_{\widehat{\beta 2}}} = \frac{-0.1585-0}{\sqrt{0.0284}}$ = -0.9405

Step 3: State decision rule

n-k = 30-2 = 28, α = 0.01

The Upper bound: β_2-($t_{\frac{\alpha}{2}}$)x($\sigma_{\widehat{\beta_2}}$) = 0 – (2.467)x($\sqrt{0.0284}$) = 0.4158



Step 4: t_{cal} = -0.9405, within boundary of CI, we can't reject null hypothesis at significance level of α = 0.01, 99%. We can't say for sure that β_2 is less than 0, 99 out of 100 times.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

From $\widehat{Y_i} = \hat{\beta}_1 + \widehat{\beta_2}X_i + \widehat{u_i}$, $\widehat{\beta_2}$ = -502.4.

When a car is 1 year(X_i) older, price(Y) decreases by 502.4 holding others constant. When vehicle is older, value decrease hence price down.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

Given α = 0.05, $\frac{\alpha}{2}$ = 0.025, n-k = 11-2 = 9

E(Y|X=5) = $\widehat{Y_0}$ = $\widehat{\beta_1} + \widehat{\beta_2}X_0$ = 7,836-502.4(5) = $5,324

Average 5 years old car is $5,324

Step 1: var($\widehat{Y_0}$) = $\sigma^2(\frac{1}{n} + \frac{(X_0-\bar{X})^2}{\sum(X_i-\bar{X})^2})$ = $212{,}877(\frac{1}{11}+\frac{(5-7.45)^2}{\sum 78.73})$ = 35,582.5345

Step 2: $\sigma_{\widehat{Y_0}}$ = $\sqrt{35{,}582.5345}$ = 188.6333

Step 3: 95% confidence interval for E(Y|X_0=5)

Upper bound: $\widehat{Y_0}$+[($t_{\frac{\alpha}{2}}$)($\sigma_{\widehat{Y_0}}$)] = 5,324+[(2.262)x(188.6333)] = $5,750.6885

Lower bound: $\widehat{Y_0}$-[($t_{\frac{\alpha}{2}}$)($\sigma_{\widehat{Y_0}}$)] = 5,324-[(2.262)x(188.6333)] = $4,887.3115

The market price range that 95% of the time that within 5 years is $4,887.3115 to $5,750.6885

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

$\widehat{Y_i}$ = 7,836-502.4X_i

se= (52) (411.8)

If X times 10

New $\widehat{\beta_1}$ = 7,836

New $\widehat{\beta_2}$ = 502.4(10) = 5024

When a car is 1 year older, the market price decrease by $502.4

se = 411.8

When a car is 10 years older, the market price decrease by $5,024

se = 4,118

d) Calculate the elasticity of market price when a car is 10 years old.

$\widehat{Y_i}$ = 7,836-502.4(10) = $2,812