Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points)

Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

From consumer portfolio choice problem in risk-free-case.

$$U'(C_0) = R_f \delta E[U'(C_1)].$$

$$\frac{1}{C_0} = R_f \delta E\left(\frac{1}{C_1}\right)$$

$$R_f = \delta^{-1} E\left(\frac{C_1}{C_0}\right) = \delta^{-1} \mu_c$$

Since $\frac{C_1}{C_0}$ reflects consumption growth. If R_f was higher, so was the consumption.

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$\sigma = \frac{R_f}{\mu_c}\frac{\partial \mu_c}{\partial R_f} = \frac{\delta^{-1}\cancel{\mu_c}}{\cancel{\mu_c}}\,\delta = 1.$$

Note : $\frac{\partial R_f}{\partial \mu_c} = \frac{\partial \delta^{-1}\mu_c}{\partial \mu_c} = \frac{1}{\delta}$

When interest rate fell, there are 2 effects.

1. Substitution effect : less return incentivise one to consume now, increasing consumption level.
2. Income effect : less return means we become poorer, so we consume less.

Since $\sigma = 1$, both effect offset each other. ∴ the consumption level will stay the same.

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

from $P_i = E\left[\frac{\delta U'(C_1)}{U'(C_0)} X_i\right]$

$P_i = E\left[\frac{\delta C_0}{C_1} X_i\right]$

The pricing kernel $m_{01} = \frac{\delta C_0}{C_1}$

Meaning: In the state where C_1 is high, m_{01} will be low. This is indicative that one does not value the return much and vice versa.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

note $M_{01} = \frac{\delta C_0}{C_1}$

$E\left(\frac{\delta C_0}{C_1}\right) = \frac{\delta}{\mu_c}$

$$\frac{\sigma_{M01}}{E(M_{01})} = \frac{E\left[\left(\frac{\delta C_0}{C_1}\right)^2\right] - \left[E\left(\frac{\delta C_0}{C_1}\right)\right]^2}{E\left(\frac{\delta C_0}{C_1}\right)}$$

$$= \left(\cancel{\frac{\delta}{\mu_c}} \cdot \cancel{\frac{\delta}{\mu_c}}\right) + \sigma_c - \cancel{\left(\frac{\delta}{\mu_c}\right)^2} \Big/ \frac{\delta}{\mu_c}$$

$$= \frac{\sigma_c}{\frac{\delta}{\mu_c}} = \frac{\sigma_c \mu_c}{\delta}$$

$$\therefore \left[\frac{E(R_i) - R_f}{\sigma_{R_i}}\right] \leq \frac{\sigma_c \mu_c}{\delta}$$

If $\exists$ a portfolio asset whose return is perfectly negatively correlated with M_{01}. The slope of the capital market line $S_e = \frac{E(R_M) - R_f}{\sigma_{R_M}} = \frac{\sigma_c \mu_c}{\delta}$.