EE 325 Answer Practice

1. In Table 1.a. $X_i$ is total microeconomics exam point (total points are 100) and $Y_i$ is GPA of each student.

**Table 1.a**

| Student | $Y_i$ | $X_i$ |
|---|---|---|
| 1 | 2.8 | 63 |
| 2 | 3.4 | 72 |
| 3 | 3 | 78 |
| 4 | 3.5 | 81 |
| 5 | 3.6 | 87 |
| 6 | 3.0 | 75 |
| 7 | 2.7 | 75 |
| 8 | 3.7 | 90 |

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 3.2125 - 0.0341(77.625) = 0.5681$$

When total microeconomics exam point is equal to zero, student's GPA is 0.5681. When total microeconomics exam point increase 1 point, student's GPA increases approximately 0.0341.

1.2 Find $\hat{Y}_i$ *and* $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

Construct the table that contain $\hat{Y}_i$ *and* $\hat{u}_i$, you will get $\sum_{i=1}^{n} \hat{u}_i = 8.327 \times 10^{-16}$

1.3 Find $\text{var}(\hat{u}_i), \text{var}(\hat{\beta}_1), and \ \text{var}(\hat{\beta}_2)$

$$\text{var}(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = \frac{0.4347253}{8-2} = 0.0725$$

$$\text{var}\left(\hat{\beta}_1\right) = \frac{\sum_{i=1}^{n} X_i^2}{n\sum_{i=1}^{n} x_i^2}\hat{\sigma}^2 = \frac{48717\left(0.0725\right)}{8\left(511.875\right)} = 0.862$$

$$\text{var}\left(\hat{\beta}_2\right) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} x_i^2} = \frac{0.0725}{511.875} = 0.00014163$$

1.4 Test the hypothesis that total microeconomics exam point has no influence on GPA at $\alpha = 5\%$

$H_0 : \beta_2 = 0,\ H_1 : \beta_2 \neq 0$

$$t = \frac{0.0341 - 0}{\sqrt{0.00014163}} = 2.863324$$

$$t_{\frac{0.05}{2},8-2} = 2.447$$

$$t = 2.863324 > t_{\frac{0.05}{2},8-2} = 2.447$$

Reject the null hypothesis. Total microeconomics exam point has influence on GPA at $\alpha = 5\%$

1.5 What percentage of the total variation in Y explained by the regression model?

$$R^2 = \frac{ESS}{TSS} = \frac{\sum_{i=1}^{8} \hat{y}_i^2}{\sum_{i=1}^{8} y_i^2} = \frac{0.59402}{1.02875} = 0.5774$$

The total variation in Y explained by the regression model is 57.74%.

1.6 Establish a 95 percent confidence interval for $E(Y \mid X = 77.6)$

$$E\left(Y \middle| X = 77.6\right) = 0.5681 + 0.0341\left(77.6\right) = 3.21$$

$$\Pr\left[\hat{\beta}_1 + \hat{\beta}_2 X_0 - t_{\frac{\alpha}{2},n-k} se\left(\hat{Y}_0\right) \leq \beta_1 + \beta_2 X_0 \leq \hat{\beta}_1 + \hat{\beta}_2 X_0 + t_{\frac{\alpha}{2},n-k} se\left(\hat{Y}_0\right)\right] = 1 - \alpha$$

$$\text{var}\left(\hat{Y} \middle| X = 77.6\right) = \sigma^2\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum_{i=1}^{n} x_i^2}\right] = 0.0725\left[\frac{1}{8} + \frac{\left(77.6 - 77.625\right)^2}{511.875}\right] = 0.0091$$

$$se\left(\hat{Y} \middle| X = 77.6\right) = \sqrt{0.0091} = 0.0952$$

$$t_{\frac{0.05}{2},8-2} = 2.447$$

$$\Pr\left[3.21 - 2.447(0.0952) \le E\left(Y\middle| X = 77.6\right) \le 3.21 + 2.447(0.0952)\right] = 0.95$$
$$\Pr\left[2.98 \le E\left(Y\middle| X = 77.6\right) \le 3.44\right] = 0.95$$

1.7 Establish a 95 percent confidence interval for $E(Y \mid X = 100)$. Compare the answer with 1.6 whether the confidence interval is wider or narrower?

$$E\left(Y\middle| X = 100\right) = 0.5681 + 0.0341(100) = 3.98$$
$$\Pr\left[\hat{\beta}_1 + \hat{\beta}_2 X_0 - t_{\frac{\alpha}{2},n-k} se\left(\hat{Y}_0\right) \le \beta_1 + \beta_2 X_0 \le \hat{\beta}_1 + \hat{\beta}_2 X_0 + t_{\frac{\alpha}{2},n-k} se\left(\hat{Y}_0\right)\right] = 1 - \alpha$$
$$\text{var}\left(\hat{Y}\middle| X = 100\right) = \sigma^2\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum_{i=1}^{n} x_i^2}\right] = 0.0725\left[\frac{1}{8} + \frac{(100 - 77.625)^2}{511.875}\right] = 0.07997$$

$$se\left(\hat{Y}\middle| X = 100\right) = \sqrt{0.0091} = 0.2828$$
$$t_{\frac{0.05}{2},8-2} = 2.447$$
$$\Pr\left[3.98 - 2.447(0.2828) \le E\left(Y\middle| X = 100\right) \le 3.98 + 2.447(0.2828)\right] = 0.95$$
$$\Pr\left[3.29 \le E\left(Y\middle| X = 77.6\right) \le 4.67\right] = 0.95$$

The confidence interval is wider than 1.6.

2. Given $Y$ is wages per hour (\$), $X$ is year of schooling (years) from a sample of 528 observations

$$\hat{Y}_t = 0.7437 + 0.6416 X_t$$
$$se = (0.8355)(0.0664)$$
$$r^2 = 0.8944, \hat{\sigma}^2 = 0.8040$$
$$\bar{X} = 12 \quad \sum x_i^2 = 2054$$

2.1 Test the hypothesis that year of schooling has a positive influence on wages per hour at $\alpha = 5\%$

$H_0 : \beta_2 \le 0,\ H_1 : \beta_2 > 0$

$$t = \frac{0.6416 - 0}{0.0664} = 9.6625 > t_{0.05, df=526} = 1.645$$

Reject the null hypothesis. Year of schooling has a positive influence on wages per hour at $\alpha = 5\%$

2.2 Interpret the regression

$\beta_2 = 0.6416$

When year of schooling increases 1 year, wages per hour ($) will increase approximately $ 0.6416.

2.3 If Miss Lily has 8 years of schooling, what is the predicted average on wages per hour ($)?

$$E\left(Y_0 \middle| X_0 = 8\right) = 5.8765$$

$$\operatorname{var}(\hat{Y}_0) = \sigma^2\left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2}\right]$$

$$\hat{\sigma}^2 = 0.8040, n = 528, \bar{X} = 12, \sum x_i^2 = 2054, X_0 = 8$$

$$\Pr\left[\hat{\beta}_1 + \hat{\beta}_2 X_0 - t_{\frac{\alpha}{2},n-k}\, se\left(\hat{Y}_0\right) \le \beta_1 + \beta_2 X_0 \le \hat{\beta}_1 + \hat{\beta}_2 X_0 + t_{\frac{\alpha}{2},n-k}\, se\left(\hat{Y}_0\right)\right] = 1 - \alpha$$

$$\Pr\left(5.6492 \le E\left(Y_0 \middle| X_0 = 8\right) \le 6.1038\right) = 0.95$$

3. Consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$ Y is Supply for good (unit: hundred pieces), X is price of good (unit: thousand baht) from a sample of 5 observation

$$\hat{Y}_i = 475.9444 - 0.4579 X_i$$

$$se = (\_\_\_\_\_\_) \quad (0.0828)$$

$$t = (8.7606) \quad (\_\_\_\_\_\_)$$

$$\hat{\sigma}^2 = 2364.7694$$

TSS = 43245.2

RSS= 7094.3080

3.1 Fill in the blank and Test the hypothesis at $\alpha = 5\%$

$se_{\beta_1} = 54.3277$

$t_{\beta_2} = -5.5294$

$H_0 : \beta_1 = 0,\ H_1 : \beta_1 \ne 0$

$t_{\beta_1} = 8.7606$

$t_{0.025,3} = 3.182$

Reject the null hypothesis.

The constant term has influence on supply for good at $\alpha = 5\%$

$H_0 : \beta_2 = 0,\ H_1 : \beta_2 \neq 0$

$t_{\beta_2} = -5.5294$

$\left| t_{\beta_2} \right| = 5.5294$

$t_{0.025,3} = 3.182$

$\left| t_{\beta_2} \right| > 3.182$ Reject the null hypothesis.
Price of good has influence on supply for good at $\alpha = 5\%$

3.2 Interpret the regression

When price of good is zero, supply for good are 475.9444 hundred pieces. When price of good increases one thousand baht, supply for good decreases 0.4579 hundred pieces.

3.3 Establish a 95 percent confidence interval for $\beta_2$ and Test the hypothesis that $\beta_2 = -0.6$ or not.

$H_0 : \beta = -0.6,\ H_1 : \beta \neq -0.6$

$\Pr[\hat{\beta}_2 - t_{\alpha/2} se(\hat{\beta}_2) \leq \beta_2 \leq \hat{\beta}_2 + t_{\alpha/2} se(\hat{\beta}_2)] = 1 - \alpha$

$\Pr[-0.4579 - 3.182(0.0828) \leq \beta_2 \leq -0.4579 + 3.182(0.0828)] = 95$

$\Pr\left(-0.7214 \leq \beta \leq -0.1944\right) = 0.95$

The confidence interval does contain -0.6. We cannot reject the null hypothesis. When price of good increases one thousand baht, supply for good decreases 0.6 hundred pieces.

3.4 What percentage of the total variation in Y explained by the regression model?

R-squared = ESS/TSS or 1-(RSS/TSS)
$R^2 = 0.8360$

3.5 If the unit Supply for good changes from hundred pieces to piece. What is the estimator for $\beta_1$ *and* $\beta_2$? Interpret the regression

$Y_i^* = w_1 Y_i,\ w_1 = 100$

$\hat{\beta}_1^* = w_1 \hat{\beta}_1 = 47594.44, \hat{\beta}_2^* = \left(\frac{w_1}{1}\right) \hat{\beta}_2 = -45.79$

$\hat{\beta}_1^* = 47594.44$ When price of good is zero, supply for good is 47594.44 pieces.

$\hat{\beta}_2^* = -45.79$ When price of good increases one thousand baht, supply for good decreases 45.79 pieces

3.6 If the unit price of good changes from thousand baht to baht. What is the estimator for $\beta_1$ *and* $\beta_2$? Interpret the regression

$X_i^* = w_2 X_i,\ w_2 = 1000$

$$\hat{\beta}_1^* = \hat{\beta}_1 = 475.9444, \hat{\beta}_2^* = \left(\frac{1}{w_2}\right)\hat{\beta}_2 = \frac{-0.4579}{1000} = -0.0004579$$

$\hat{\beta}_1^* = 475.9444$ When price of good is zero, supply for good is 475.9444 hundred pieces.

$\hat{\beta}_2^* = -0.0004579$ When price of good increases one baht, supply for good decreases 0.0004579 hundred pieces

3.7 If the unit Supply for good changes from hundred pieces to piece and the unit price of good changes from thousand baht to baht. What is the estimator for $\beta_1$ *and* $\beta_2$? Interpret the regression

$Y_i^* = w_1 Y_i,\ w_1 = 100\ \ X_i^* = w_2 X_i,\ w_2 = 1000$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 47594.44, \hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = -0.04579$$

$\hat{\beta}_1^* = 47594.44$ When price of good is zero, supply for good is 47594.44 pieces.

$\hat{\beta}_2^* = -0.04579$ When price of good increases one baht, supply for good decreases 0.04579 pieces