Answer: Exercise 3

Derivatives of trigonometric/inverse/exponential/logarithmic/hyperbolic functions

.

1. (a) $\frac{1}{\left(\sqrt{1-(x/x+1)^2}\right)(x+1)^2}$

   (b) $\left(\frac{-18x^2}{1+4x^6}\right) - x\csc^2(x) + \cot(x)$

   (c)
$$f'(x) = \frac{-\cos^{-1}(x)\sin(x) + \cos(x)\frac{1}{\sqrt{1-x^2}}}{(\cos^{-1}(x))^2}.$$

   (d) $\frac{\cos(\frac{x}{2})}{2(1+\sin^2(\frac{x}{2}))}$

   (e) $frac1\ln(2)\frac{1}{\ln(\ln(x))}\frac{1}{\ln(x)}\frac{1}{x}$

   (f) $\frac{\cos(x)}{\sqrt{1+\sin^2(x)}} + \frac{\sinh(x)}{\sqrt{1-\cosh^2(x)}}$

2. (a) $\frac{y2^{x/y}\ln(2)-2xy^2}{2y^3+x2^{x/y}\ln(2)}$

   (b) $\frac{xy\cos(e^{xy})e^{xy}+\sin(e^{xy})}{1-x^2\cos(e^{xy})e^{xy}}$

   (c) $3x^2e^{e^{x^3}}e^{x^3}$

   (d) $e^x + ex^{(e-1)}$

   (e) $\frac{3^{x+y}\ln(3)+3x^2+\tan(x)}{1-3^{x+y}\ln(3)}$

   (f) $\frac{1}{\ln(3)} + \log_3(x) + \frac{2}{x}$

   (g) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

3. $\frac{1}{\sqrt{3}}$

4. $y = \left(\frac{1}{2} + \frac{\pi}{4}\right)x - \frac{1}{2}$

5. $(8x^3 + 12x)e^{x^2}$

6. These points on the graph are $(n\pi, e^{\cos(n\pi)})$ where n is any integer, i.e $\ldots, (-2\pi, e), (-\pi, \frac{1}{e}), (0, e), (\pi, \frac{1}{e}), (2\pi, e), \ldots$:
$$\left\{(x,y) = (n\pi, e^{\cos(n\pi)}) = \begin{cases} (n\pi, e), & \text{if } \quad n = \text{ even integers } (n = 0, \pm 2, \pm 4, \ldots) \\ (n\pi, \frac{1}{e}), & \text{if } \quad n = \text{ odd integers } (n = \pm 1, \pm 3, \pm 5, \ldots) \end{cases}\right\}.$$

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8. (a) $x^{\sin(x)}\left[\sin(x)\frac{1}{x} + \ln(x)\cos(x)\right]$.

   (b) $[\sin(x)]^x\left[\frac{x\cos(x)}{\sin(x)} + \ln(\sin(x))\right]$

   (c) $x(x-e)^x\left[\frac{1}{x} + \frac{x}{x-e} + \ln(x-e)\right]$

   (d) $\frac{(x+1)^{0.1}(x+2)^{0.2}(x+3)^{0.3}}{(x^2+4)^{0.4}(x^2+5)^{0.5}}\left[\frac{0.1}{x+1} + \frac{0.2}{x+2} + \frac{0.3}{x+3} - \frac{0.8x}{x^2+4} - \frac{x}{x^2+5}\right]$.

9. (a) $\frac{dy}{dx} = x^{\sin(x)}\left[\sin(x)\frac{1}{x} + \ln(x)\cos(x)\right] + [\sin(x)]^x\left[\frac{x\cos(x)}{\sin(x)} + \ln(|\sin(x)|)\right]$
   $+ \cos(x^x)\left[1 + \ln(x)\right]x^x$

(b) $\frac{dy}{dx} = (x^x + 1)e^{x^x}\left[1 + \ln(x)\right]x^x$

(c) $\frac{dy}{dx} = (x^x + 1)e^{x^x}\left[1 + \ln(x)\right]x^x$

(d) $\frac{dy}{dx} == \frac{1}{\tanh^{-1}(e^{2x})}\frac{2e^{2x}}{(1-e^{4x})}$

10. $\frac{d^2y}{dx^2} = \frac{x^{x/3}}{3}\left(\frac{1}{x} + \frac{1}{3}\left[1 + \ln(x)\right]^2\right)$