Probability and Statistics

- The probability of an event is the long-run relative frequency of the event, given an infinite number of trials with no changes in the underlying conditions.
- Events and probabilities can be summarized through a probability distribution
  - Distributions may be discrete or continuous
- A probability distribution is characterized by:
  - A mean, or measure of central tendency
  - A variance, or measure of dispersion

Probability and Statistics

- The mean or expected value is:

$$\mu \; or \; EV = \sum X_i P_i$$

Amount of Loss (X_i)		Probability of Loss (P_i)		$X_i P_i$
\$ 0	X	0.30	=	\$ 0
\$360	X	0.50	=	\$180
\$600	X	0.20	=	\$120
		$\sum X_i P_i$	=	\$300

Probability and Statistics

- The variance of a probability distribution is:

$$\sigma^2 = \sum P_i \left(X_i - EV\right)^2$$

- For the previous loss distribution,

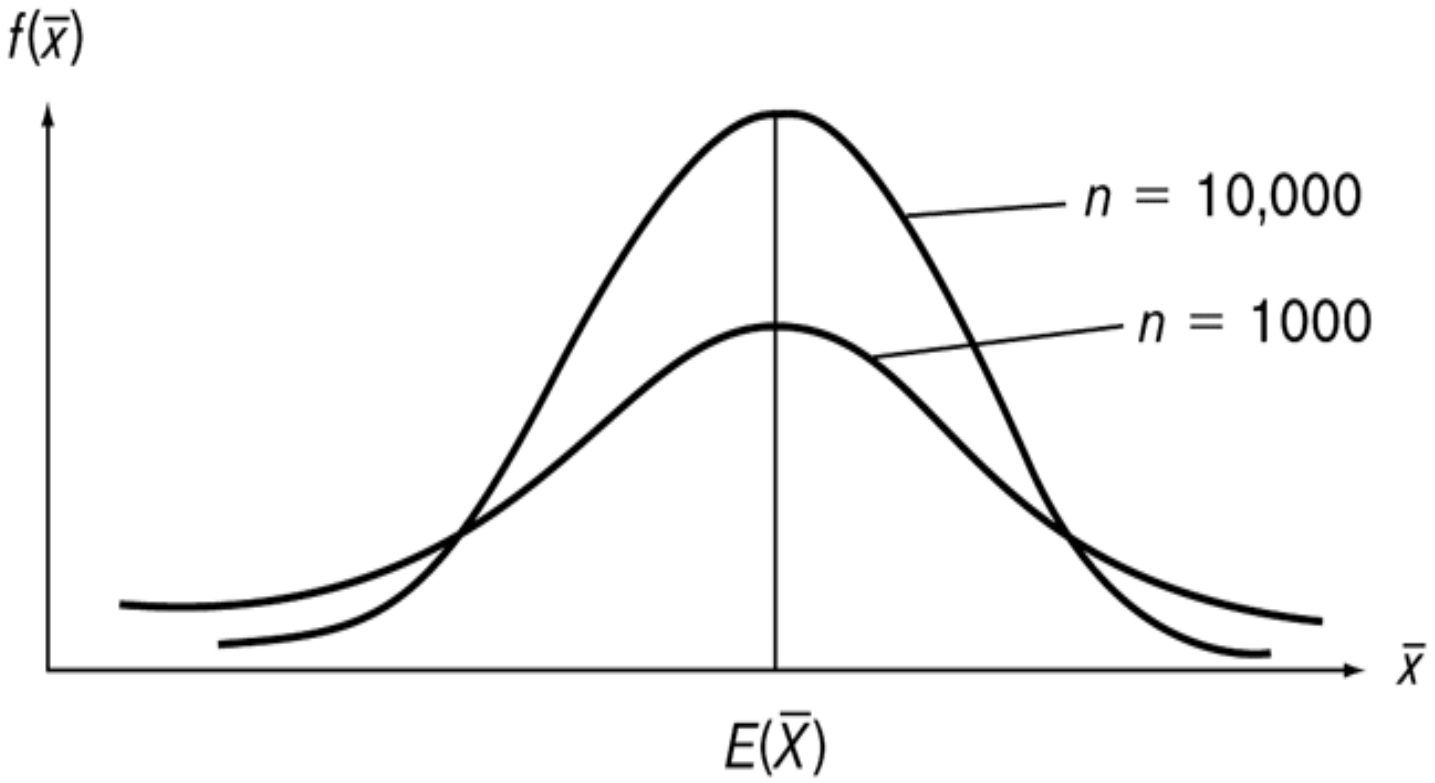
$$\begin{aligned}\sigma^2 &= 0.30(0-300)^2 + 0.50(360-300)^2 \\ &\quad +0.20(600-300)^2 \\ &= 27{,}000 + 1{,}800 + 1{,}800 \\ &= 46{,}800\end{aligned}$$

- The standard deviation = $\sqrt{\sigma^2} = \sigma = 216.33$
- Higher standard deviations, relative to the mean, are associated with greater uncertainty of loss; therefore, the risk is greater

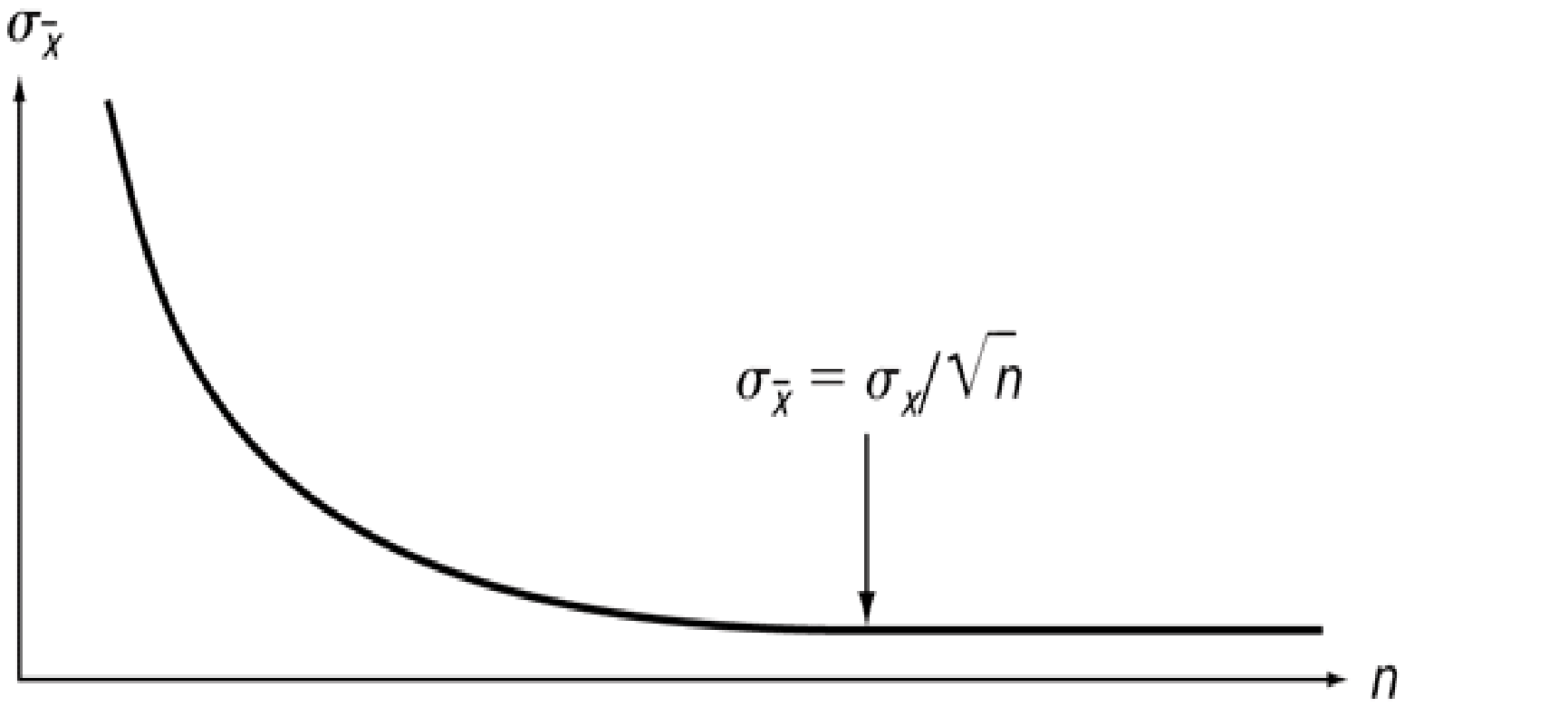
Law of Large Numbers

- The law of large numbers is the mathematical foundation of insurance.
- Average losses for a random sample of *n* exposure units will follow a normal distribution because of the Central Limit Theorem.
  - Regardless of the population distribution, the distribution of sample means will approach the normal distribution as the sample size increases.
  - The standard error of the sampling distribution can be reduced by simply increasing the sample size

Sampling Distribution Versus Sample Size



Standard Error of the Sampling Distribution Versus Sample Size



Law of Large Numbers

- When an insurer increases the size of the sample of insureds:
  - Underwriting risk increases, because more insured units could suffer a loss.
  - But, underwriting risk does not increase proportionately. It increases by the square root of the increase in the sample size.
  - There is “safety in numbers” for insurers!