1. Two individuals agree at date 0 to a forward contract that matures at date 2. The contract is written on an underlying asset that pays a dividend at date 1 equal to D_1. Let f_2 be the date 2 random payoff (profit) to the individual who is the long party in the forward contract. Also let m_{0i} be the stochastic discount factor over the period from dates 0 to i where $i = 1, 2$, and let $E_0[\cdot]$ be the expectations operator at date 0. What is the value of $E_0[m_{02}f_2]$? Explain your answer.

From $f_2 = S_2 - F_{02}$; $E_0[M_{02}f_2] = E[M_{02}S_2] - E[M_{02}F_{02}]$

$= S_0 - D_0 - R_f^{-2}F_{02}$

with absence of arbitrage $F_{02} = R_f^2(S_0 - D_0)$

$\therefore$ $E_0[M_{02}f_2] = 0$

2. Assume that there is an economy populated by infinitely-lived representative individuals who maximize the lifetime utility function

$$E_0\left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t}\right]$$

where c_t is consumption at date t and $a > 0$, $0 < \delta < 1$. The economy is a Lucas (1978) endowment economy having multiple risky assets paying date t dividends that total d_t per capita. Write down an expression for the equilibrium per capita price of the market portfolio in terms of the assets' future dividends.

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{U_c(c_t, t)}{U_c(c_0, 0)} d_t\right]$$

$$= E_0\left[\sum_{t=1}^{\infty} \delta^t e^{-a(d_t - d_0)} d_t\right]$$

3. For the Lucas model with labor income, show that assumptions (6.25) and (6.26) lead to the pricing relationship (6.27) and (6.28).

4. Consider a special case of the model of rational speculative bubbles discussed in this chapter. Assume that infinitely-lived investors are risk-neutral and that there is an asset paying a constant, one-period risk-free return of $R_f = \delta^{-1} > 1$. There is also an infinitely-lived risky asset with price p_t at date t. The risky asset is assumed to pay a dividend of d_t which is declared at date t and paid at the end of the period, date $t+1$. Consider the price $p_t = f_t + b_t$ where

$$f_t = \sum_{i=0}^{\infty} \frac{E_t\left[d_{t+i}\right]}{R_f^{\,i+1}} \tag{1}$$

and

$$b_{t+1} = \begin{cases} \frac{R_f}{q_t} b_t + e_{t+1} & \text{with probability } q_t \\ z_{t+1} & \text{with probability } 1 - q_t \end{cases} \tag{2}$$

where $E_t\left[e_{t+1}\right] = E_t\left[z_{t+1}\right] = 0$ and where q_t is a random variable as of date $t-1$ but realized at date t and is uniformly distributed between 0 and 1.

4.a Show whether or not $p_t = f_t + b_t$ subject to the specifications in (1) and (2) is a valid solution for the price of the risky asset.

$$E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] + (1-q_t)E_t[z_{t+1}] = R_f b_t$$

thus is valid.

4.b Suppose that p_t is the price of a barrel of oil. If $p_t \geq p_{solar}$, then solar energy, which is in perfectly elastic supply, becomes an economically efficient perfect substitute for oil. Can a rational speculative bubble exist for the price of oil? Explain why or why not.

4.c Suppose p_t is the price of a bond that matures at date $T < \infty$. In this context, the d_t for $t \leq T$ denotes the bond's coupon and principal payments. Can a rational speculative bubble exist for the price of this bond? Explain why or why not.

b.

c. Bubble cannot exist because, at maturity, bond price must be $P_t = d_t$ and 0 after T. The price cannot be expected to increase indefinitely because it is invalid. The only rational price is $P_t = P_t^*$.

5. Consider an endowment economy with representative agents who maximize the following objective function:

$$\max_{C_s, \{\omega_{is}\}, \forall s, i} E_t \left[\sum_{s=t}^{T} \delta^s u(C_s) \right]$$

where $T < \infty$. Explain why a rational speculative asset price bubble could not exist in such an economy.