Homework 3

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1. Find Cournot equilibrium when there are 3 firms in the market $P = a - bQ$; $Q = q_1 + q_2 + q_3$ $c = c_1 = c_2 = c_3$ what is equilibrium price ? : P^*

what are firms' profit ? $\pi_1 = \pi_2 = \pi_3 = ?$ $\pi_i = Pq_i - Cq_i = [a - b(q_1 + q_2 + q_3)]q_i - cq_i$

firm 1 ; $\pi_1 = TR_1 - TC_1$

$\pi_1 = (a - bq_1 - bq_2 - bq_3)q_1 - C_1q_1$ ($aq_1 - bq_1^2 - bq_1q_2 - bq_1q_3$)

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 - c_1 = 0$

$a - bq_2 - bq_3 - c_1 = 2bq_1$

BR of firm 1 $q_1^* = \frac{a - bq_2 - bq_3 - c}{2b}$

substitute 2 ; $q_1^* = a - b\left(\frac{a - bq_1 - bq_3}{2b}\right) - q_3 b$ / $2b$

$q_1^* = \frac{a - q_3 b}{3b}$ —— (1)

substitute 3 ; $q_1 = a - \left(\frac{a}{4b}\right)b$ / $3b$

$q_1^* = \frac{4a - a}{12b} = \frac{a}{4b}$

firm 2 ; $\pi_2 = TR_2 - TC_2$

$\pi_2 = (a - bq_1 - bq_2 - bq_3)q_2 - c_2q_2$ ($aq_2 - bq_1q_2 - bq_2^2 - bq_2q_3$)

$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 - c_2 = 0$

$a - bq_1 - bq_3 - c_2 = 2bq_2$ $\therefore \frac{a - bq_1 - bq_3 - c}{2b} = q_2^*$

substitute 1 ; $2q_2 b = a - \left(\frac{a - q_2 b - q_3 b}{2b}\right)b - q_3 b$

$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$

$4q_2 b = a + q_2 b - q_3 b$

$3q_2 b = a - q_3 b$

$q_2^* = \frac{a - q_3 b}{3b}$ —— (2)

substitute 3 ; $q_2 = \frac{a - \left(\frac{a}{4b}\right)b}{3b}$

$q_2^* = \frac{4a - a}{12b} = \frac{a}{4b}$

firm 3 ; $\pi_3 = TR_3 - TC_3$

$\pi_3 = (a - bq_1 - bq_2 - bq_3)q_3 - c_3q_3$ ($aq_3 - bq_1q_3 - bq_2q_3 - bq_3^2$)

$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 - c = 0$

$a - bq_1 - bq_2 - c = 2bq_3$

$q_3^* = \frac{a - bq_1 - bq_2 - c}{2b}$

substitute 1 and 2 $q_3^* = \frac{a - b\left(\frac{a - q_3 b}{3b}\right) - b\left(\frac{a - q_3 b}{3b}\right)}{2b}$

$q_3^* = \frac{3a - a + q_3 b - a + q_3 b}{6b}$

$q_3^* = \frac{a + 2q_3 b}{6b}$

$6bq_3 = a + 2q_3 b$

$4bq_3 = a + 2q_3 b$

$4bq_3 = a$

$q_3^* = \frac{a}{4b}$

Equilibrium price ; $P = a - bQ$

$P = a - b(q_1 + q_2 + q_3)$

$P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$

$P = a - \left(\frac{3a}{4}\right)$

$P = \frac{a}{4} = 0.25a$

firm 1 ; $\pi_1 = P \cdot q_1 - c_1$

$\pi_1 = 0.25a \cdot \frac{a}{4b} - c_1$

$\pi_1 = \frac{a^2}{16b} - c_1$ #

Firm 2 ; $\pi_2 = P \cdot q_2 - c_2$

$\pi_2 = 0.25a \cdot \frac{a}{4b} - c_2$

$\pi_2 = \frac{a^2}{16b} - c_2$ #

Firm 3 ; $\pi_3 = P \cdot q_3 - c_3$

$\pi_3 = 0.25a \cdot \frac{a}{4b} - c_3$

$\pi_3 = \frac{a^2}{16b} - c_3$ #

2. If there are N firm $q_i^* = f(N)$, $P = f(N)$, $\pi_i = f(N)$

assume $q_1 + q_2 + q_3 + q_n = A$

$P = a - b(q_1 + q_2 + q_3 + \dots + q_n)$

$P = a - bq_1 - bq_2 - \dots - bq_n$

$\pi_1 = (a - bq_1 - bq_2 - bq_3 - \dots - bq_n) \cdot q_1 - c_1$

$\vdots$

$\pi_n = (a - bq_1 - bq_2 - bq_3 - \dots - bq_n) \cdot q_1 - c_n$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 - \dots - bq_n = 0$

$q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$

$\vdots$

$q_n = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \dots + q_{n-1})$

$q_1^* = q_2^* = \dots = q_n^* = \frac{a}{b} - A$ —— (1)

$\therefore q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \dots + q_n)$

$0.5q_1 = \frac{a}{2b} - 0.5A$

$q_1 = \frac{a}{b} - A$ —— (1)

$q_2 = \frac{a}{b} - A$

$q_3 = \frac{a}{b} - A$

$\vdots$

$q_n = \frac{a}{b} - A$

since $A = q_1 + q_2 + q_3 + \dots q_n$, $A = n\left(\frac{a}{b} - A\right)$

$A = n\frac{a}{b} - nA$

$A + nA = n\frac{a}{b}$

$A(1 + n) = n\frac{a}{b}$

$A = \frac{na}{(n+1)b}$ ⟶ sub into (1) ; $q_1 = \frac{a}{(n+1)b}$

$q_1 = \frac{a}{(n+1)b}$

Equilibrium Price ; $P = a - b(A)$

$P = a - b\left(\frac{na}{(n+1)b}\right)$

$P = a - \left(\frac{n}{n+1}\right)a$

$P = \frac{a(n+1) - na}{n+1}$

$P = \frac{na + a - na}{n+1}$

$P = \frac{a}{n+1}$

$\pi_i = P \cdot q_i - c_i$

$\pi_i = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - c_i$

$\pi_i = \frac{a^2}{(n+1)^2 b} - c_i$

3. from question 2, what happen if $N \to \infty$ and what happen if $N=1$

If $N \to \infty$ which mean that the market output (Q) goes to a competitive level and the price (P^*) converges to marginal cost. The market is perfectly competitive market. From cournot model, there must be 2 or more firms competing in the market. So, if $n=1$ firm becomes a monopolist.

if $n \to \infty$, it will make $q_i = \frac{a}{(n+1)b} \longrightarrow$ nearly zero and each firms will sell at q_i nearly zero unit

it will make $A = nq_i \longrightarrow$ nearly zero . Q of every firms combinded will be nearly $\to a$ unit

it will make $P = \frac{a}{n+1} \longrightarrow$ nearly zero . When supply increase, price will decrease $\to$ nearly zero

it will make $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i \longrightarrow -c_i$ Each firm will lose the profit

if $n=1$, it will make $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ .Since $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$, monopoly will sell less quantity

it will make $A = nq_i = Q$. Since $n=1$, the firm will be a Monopoly

it will make $P = \frac{a}{n+1} = \frac{a}{2}$. Since $P_M = \frac{a}{2} > P = \frac{a}{n+1}$, Monopoly will set higher price

it will make $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$, which higher than $\pi_i = \frac{a^2}{(n+1)^2 b}$, Monopoly will get higher profit.