

## 4 Testing for Serial Correlation

Given the model

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \dots + \beta_k x_{tk} + u_t$$

### 4.1 A "t-test" for AR(1) serial correlation with strictly exogenous regressors

The most common type of serial correlation or autocorrelation is the AR(1) type:

To perform the test:

1. Estimate  $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \dots + \beta_k x_{tk} + u_t$
2. Obtain  $\hat{u}_t, \hat{u}_{t-1}; \forall t = 1, 2, \dots, n$
3. Estimate  $\hat{u}_t = \rho \hat{u}_{t-1} + error$

4. Perform the  $t$  – test for

4.2 *The Durbin-Watson Test (DW test)*

This implies

$$\hat{\rho} = 0 \Rightarrow DW = 2$$

$$\hat{\rho} > 0 \Rightarrow DW < 2$$

$$\hat{\rho} < 0 \Rightarrow DW > 2$$

$H_o$  : no positive autocorrelation, serial-correlation

$H_a$  : no negative serial correlation

To perform the test:

1. Estimate  $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \dots + \beta_k x_{tk} + u_t$
2. Obtain  $\hat{u}_t, \hat{u}_{t-1}; \forall t = 1, 2, \dots, n$
3. Calculate  $DW$  from eq.(2)
4. Find the critical  $d_L$  and  $d_u$  values (say, at the 5% level of significance) for the given sample size and # of regressors.
5. Follow the decision rule in the picture.

Example:

Suppose the calculated value of  $DW = 0.80$ ,  $n = 45$ ,  $k = 4$ .

From this, we get  $d_L = \text{-----}$  and  $d_u = \text{-----}$

#### 4.3 Testing for $AR(1)$ serial correlation "without" strictly exogenous regressors

4.4 *Testing for AR(q) serial correlation "without" strictly exogenous regressors*

5 Correcting for serial correlation

5.1 *Passive way*

Use the type of standard error that is robust to the serial correlation, autocorrelation problem

5.2 *Active way –*