<u>EE320 Placement test</u>

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

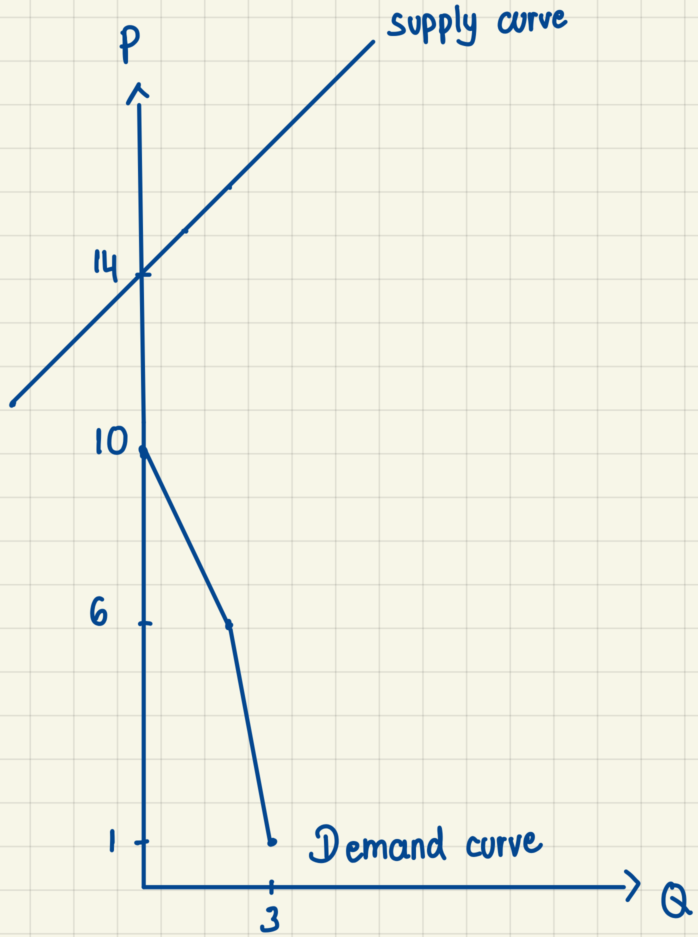
1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If “a” increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

Answer

1.1. Graph the market demand and market supply

$D: P = 10 - Q^2$

$S: Q = -14 + P \Rightarrow P = Q + 14$



1.2. With the given supply and demand equations, there is no equilibrium in this market. This is because the minimum price required by producer is 14 while the maximum price that buyers are willing to pay is 10. Thus, there is no equilibrium.

1.3. If $a = -12$, there is still no equilibrium exists in this market.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

Answer

$$R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right)$$

$$R'(Q) = \frac{1}{Q^2+1}(2Q) + 3\left[\frac{Q+1-Q}{(Q+1)^2}\right]$$

$$= \frac{2Q}{Q^2+1} + 3\,\frac{1}{(Q+1)^2} = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

Therefore, marginal revenue function is $\boxed{R'(Q) = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}}$

Set $R'(x) = 0$

$$\Rightarrow \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2} = 0$$

$$\frac{2Q(Q+1)^2 + 3(Q^2+1)}{(Q^2+1)(Q+1)^2} = 0$$

Since $(Q^2+1)(Q+1)^2$ always >0, the sign of $f'(x)$ depends on the numerator

$$2Q(Q+1)^2 + 3(Q^2+1) = 0$$

$$2Q(Q+1)^2 + 3(Q+1)(Q-1) = 0$$

$$(Q+1)[2Q(Q+1) + 3(Q-1)] = 0$$

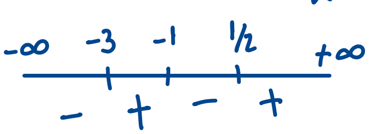
$$\Rightarrow Q = -1$$

$$\Rightarrow 2Q^2 + 2Q + 3Q - 3 = 0$$

$$2Q^2 + 5Q - 3 = 0$$

$$\Delta = 5^2 - 4(2)(-3) = 25 + 24 = 49$$

$$\Rightarrow Q = \frac{-5 \pm \sqrt{49}}{2 \cdot 2} = \begin{cases} \frac{-5+7}{4} = \frac{1}{2} \\ \frac{-5-7}{4} = -\frac{12}{4} = -3 \end{cases}$$



Therefore, revenue function increases at $Q \in (-3, -1) \cup (\frac{1}{2}, +\infty)$ and decreases when $Q \in (-\infty, -3) \cup (-1, \frac{1}{2})$.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

Answer

$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$

$\pi'(Q) = -Q^2 - 2Q + 8$

$\pi'(Q) = 0 \iff -Q^2 - 2Q + 8 = 0$

$-(Q^2 + 2Q - 8) = 0$

$-(Q-2)(Q+4) = 0 \Rightarrow Q = 2, -4$

Sign of $\pi'(Q)$

	$-\infty$		-4		2		$+\infty$
$\pi'(Q)$		$-$	0	$+$	0	$-$	
$\pi(Q)$		$\searrow$		$\nearrow$	$\pi(2)$	$\searrow$	

$\therefore$ Therefore, when $Q = 2$, $\pi(Q)$ will be maximized.

When $Q = 2$, $\pi(2) = -\frac{1}{3}(2)^3 - 2^2 + 8\cdot 2 - 1 = -\frac{8}{3} - 4 + 16 - 1 = 11 - \frac{8}{3} = \frac{25}{3}$

second derivative

$\pi''(Q) = -2Q - 2$

$\pi''(2) = -2(2) - 2 = -4 - 2 = -6 < 0 \Rightarrow \pi(Q)$ has relative maximization at $Q = 2$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

A+B is undefined because the two maxtrices have different dimensions.

4.2 A*B

$$A \times B = \underset{2\times2}{A} \times \underset{2\times3}{B} = \begin{bmatrix} 8\cdot1+9\cdot4 & 8\cdot2+9\cdot5 & 8\cdot3+9\cdot6 \\ 10\cdot1+11\cdot4 & 10\cdot2+11\cdot5 & 10\cdot3+11\cdot6 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$$\det(A) = (8\times11) - (9\times10) = -2$$

4.4 det(B)

det(B) is not possible because B is not a square matrix.

4.5 det(C)

$$\det(C) = 1\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} - 2\begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + 3\begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$$

$$= -3 + 12 - 9 = 0$$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

Answer

- Calculate $\frac{\partial U}{\partial x}$

$$\frac{\partial U}{\partial x} = axy^b + \frac{1}{\frac{x}{x+y}} \frac{d}{dx}\left(\frac{x}{x+y}\right) = axy^b + \frac{x+y}{x} \cdot \frac{x+y-x}{(x+y)^2} = \boxed{axy^b + \frac{xy+y^2}{x(x+y)^2}}$$

- Calculate $\frac{\partial U}{\partial y}$

$$\frac{\partial U}{\partial y} = byx^a + \frac{x+y}{x} \frac{d}{dy}\left(\frac{x}{x+y}\right) = byx^a + \frac{x+y}{x} \cdot \frac{(-x)}{(x+y)^2} = \boxed{byx^a + \frac{(-x^2-xy)}{x(x+y)^2}}$$