

EE441: Economics of Public Expenditure

Cost-Benefit Analysis



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Introduction

- ***Cost-benefit analysis*** is a set of practical procedures for guiding public expenditure decisions.

Present Value

- Project evaluation usually requires comparing costs and benefits from different time periods
- Dollars across time periods are not immediately comparable, because of inflation and returns in the market.

Present Value: Present Dollars into the Future

- Suppose you invest \$100 today in the bank . . .
 - At the end of year 1, it is worth $(1+.05)\times\$100$, or \$105
 - At the end of year 2, it is worth $(1+.05)\times\$105$, or \$110.25
 - The interest compounds over time, that is, the interest is also earning interest.

Present Value: Present Dollars into the Future

- Define
 - R = initial investment amount
 - r = rate of return on investment
 - T = years of investment
- The future value (FV) of the investment is:

$$FV = R(1+r)^T$$

EX: If $T=2$, $FV = R(1+r)^2$

Present Value: Future Dollars into the Present

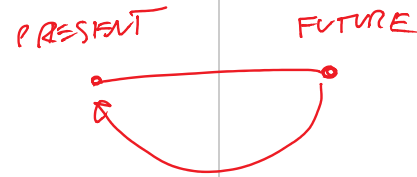
- Suppose someone promises to pay you \$100 *one year from now*.
- What is the maximum amount you should be willing to pay today for such a promise?
- You are forgoing the interest that you could earn on the money that is being loaned.

Present Value: Future Dollars into the Present

- The ***present value*** of a future amount of money is the maximum amount you would be willing to pay today for the right to receive the money in the future.

Present Value: Present Dollars into the Future

- Define
 - R = amount to be received in future
 - r = rate of return on investment
 - T = years of investment
- The present value (PV) of the investment is:



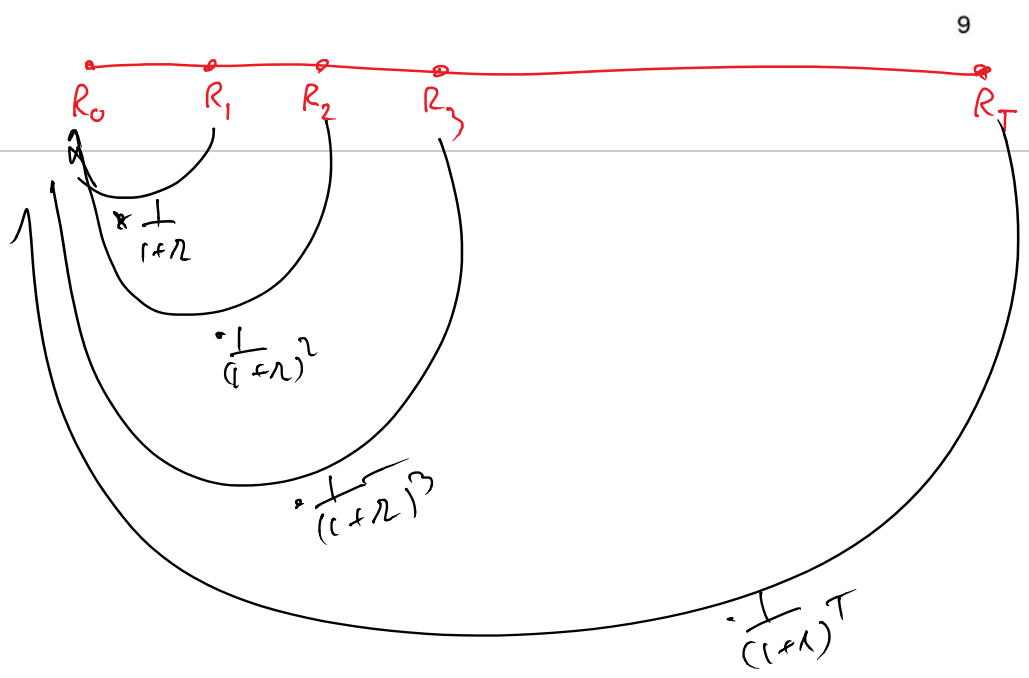
$$PV = \frac{R}{(1+r)^T}$$

Handwritten annotations: A red line points from the text "DISCOUNT RATE" to the r in the denominator. A bracket under the denominator is labeled "DISCOUNT FACTOR".

Present Value: Future Dollars into the Present

- In previous equation, r is often referred to as the **discount rate**, and $(1+r)^{-T}$ is the **discount factor**.
OR $\frac{1}{(1+r)^T}$
- Finally, consider a promise to pay a stream of money, $\$R_0$ today, $\$R_1$ one year from now, and so on, for T years.

$$PV = R_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \frac{R_2}{(1+r)^3} + \dots + \frac{R_T}{(1+r)^T}$$



Present Value: Future Dollars into the Present

- Present value is an enormously important concept
- A \$1,000,000 payment 20 years from now is only worth today:
 - \$376,889 if $r=.05$ $r = 5\%$.
 - \$148,644 if $r=.10$ $r = 10\%$.

THE HIGHER THE DISCOUNT RATE, THE LOWER PRESENT
VALUE 10

Present Value: Inflation

- **Nominal amounts** are valued according to the level of prices in the year the return occurs.
- **Real amounts** are valued according to the level of prices in one particular year.
- Inflation affects both the payout stream and the discount factor, and these two cancel each other out.

$$PV = R_0 + \frac{\cancel{(1+\pi)} R_1}{\cancel{(1+\pi)}(1+r)} + \frac{\cancel{(1+\pi)}^2 R_2}{\cancel{(1+\pi)}^2(1+r)^2} + \frac{\cancel{(1+\pi)}^3 R_3}{\cancel{(1+\pi)}^3(1+r)^3} + \dots + \frac{\cancel{(1+\pi)}^T R_T}{\cancel{(1+\pi)}^T(1+r)^T}$$

Private Sector Project Evaluation

- Suppose there are two projects, X and Y .
- Each entails certain benefits and costs, denoted as B^X , C^X , B^Y , and C^Y .

- Need to ask:

- Is the project *admissible*?
- Is the project *preferable*?

IMPLEMENT
OR
NOT IMPLEMENT

Private Sector Project Evaluation

- Admissible: Are the benefits greater than the costs?
- Preferable: Are the net benefits the highest?
- Most projects involve a stream of benefits and costs over time.

Private Sector Project Evaluation

- Define:

$B_t^i =$ Benefits from project i at time t

$C_t^i =$ Costs from project i at time t

- Then the present value of project i is:

$$PV^i = (B_0^i - C_0^i) + \frac{(B_1^i - C_1^i)}{(1+r)} + \frac{(B_2^i - C_2^i)}{(1+r)^2} + \dots + \frac{(B_T^i - C_T^i)}{(1+r)^T}$$

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Private Sector Project Evaluation

- The ***present value criteria*** for project evaluation are that:
 - A project is admissible only if its present value is positive.
 - When two projects are *mutually exclusive*, the preferred project is the one with the highest present value.

Private Sector Project Evaluation

- Table 11.2 shows two different projects (R&D or Advertising).
- The discount rate plays a key role in deciding what project to choose, because the cash inflows occur at different times.
- The lower the discount rate, the more valuable the back-loaded project.

Table 11.2

Table 11.2 Comparing the present values of two projects

Year	Annual Net Return		$r =$	PV	
	R&D	Advertising		R&D	Advertising
0	-\$1,000	-\$1,000	0	\$150	< \$200
1	600	-0-	.01	128	< 165
2	-0-	-0-	.03	86	< 98
3	550	1,200	.05	46	37
			.07	10	-21

#1 WHEN $r < 0.7$, BOTH ARE ADMISSIBLE, W/ $r = .07$, ADV IS NOT ADMISSIBLE

#2 WHEN $r = 0, .01, .03$, ADV PROJECT IS PREFERRED SINCE IT GIVES THE HIGHER PV COMPARED TO R&D PROJECT.

WHEN $r = .05, .07$, IT TURNS OUT THAT R&D IS PREFERRED TO ADV. PROJECT.

Private Sector Project Evaluation

- Several other criteria are often used for project evaluation, but can give misleading answers
 - Internal rate of return (IRR)
 - Benefit-cost ratio (B-C RATIO)

Private Sector Project Evaluation

- The **internal rate of return**, ρ , is defined as the ρ that solves the equation:

$$0 = (B_0 - C_0) + \frac{(B_1 - C_1)}{(1+\rho)} + \dots + \frac{(B_T - C_T)}{(1+\rho)^T}$$

- The IRR is the discount rate that would make the present value of the project equal to zero.
 - Admissible if $\rho > r$.
 - The flawed analysis would choose an admissible project with the higher internal rate of return, ignoring scale.

EX.

PROJECT X REQUIRES THE EXPENDITURE OF 100 \$ TODAY
AND GIVES 110 \$ A YEAR FROM NOW.

SO IRR = 10 PERCENT

PROJECT Y REQUIRES THE EXPENDITURE OF 1000 \$ TODAY
AND GIVES 1080 \$ A YEAR FROM NOW.

SO IRR = 8 PERCENT

ASSUME THAT FIRM CAN BORROW AND LEND FREELY AT
A 6 PERCENT RATE OF INTEREST.

• IF LOOKING AT IRR, X IS PREFERRED TO Y.

• HOWEVER, PROFITS FROM PROJECT X = $10 - 6 = 4$

PROFITS FROM PROJECT Y = $\frac{80 - 60}{20} = 20$

SO, THE FIRM SHOULD CHOOSE PROJECT Y!

(CONTRAST TO THE IMPLICATION FROM IRR)

* REMARK: IRR IGNORES SCALE OF THE PROJECTS AND
MIGHT LEAD TO A WRONG DECISION.

Private Sector Project Evaluation

- The **benefit-cost ratio** divides the discounted stream of benefits by the discounted stream of costs. In this case:

B=stream of benefits and C=stream of costs:

$$\text{PV of BENEFITS} \rightarrow B = B_0 + \frac{B_1}{1+r} + \dots + \frac{B_T}{(1+r)^T}$$

$$\text{PV of COSTS} \rightarrow C = C_0 + \frac{C_1}{1+r} + \dots + \frac{C_T}{(1+r)^T}$$

Private Sector Project Evaluation

- Admissibility using the *benefit-cost ratio* requires:

$$\frac{B}{C} > 1$$

- This ratio is virtually useless for comparing across admissible projects, however.
- Ratio can be manipulated by counting benefits as "negative costs" and vice-versa.

EX:

PROJECT A

PROJECT B

$$B = 250$$

$$C = 100$$

$$\frac{B}{C} = \frac{250}{100} = 2.5 > 1$$

$$B = 200$$

$$C = 100$$

$$\frac{B}{C} = \frac{200}{100} = 2.0 > 1$$

SINCE $\left(\frac{B}{C}\right)_A > \left(\frac{B}{C}\right)_B$, THEN A IS PREFERRED TO B,

SUPPOSE, IN DOING PROJECT A, IT WILL CREATE A DAMAGE OF 40 TO SOME GROUP OF PEOPLE.

SO, "40" CAN BE VIEWED AS "A REDUCTION IN BENEFITS OF PROJECT A".

AS A RESULT, $\left(\frac{B}{C}\right)_A = \frac{250 - 40}{100} = \frac{210}{100} = 2.1$.

COSTS ARE VIEWED AS NEGATIVE BENEFITS!

A IS STILL PREFERRED TO B AS $\left(\frac{B}{C}\right)_A = 2.1 > \left(\frac{B}{C}\right)_B = 2.0$

HOWEVER, IF WE REGARD "40" AS AN INCREASE IN COST OF DOING A,

THE $\left(\frac{B}{C}\right)_A$ WOULD BE $\frac{250}{100 + 40} = \frac{250}{140} = 1.79$.

NOW, B IS PREFERRED TO A.

W/ THIS FLAW → NBV PRESENT VALUE APPROACH IS
PREFERABLE AS IT COMPARES "DIFFERENCES"
BETWEEN COSTS AND BENEFITS RATHER
THAN "RATIO".

Discount Rate for Government Projects

- Government decision making involves present value calculations.
- Costs, benefits, and discount rates are somewhat different from private sector.

IF MARKET IS COMPETITIVE → NO MARKET DISTORTIONS
 NO EXTERNALITIES,
 (SOCIAL DISCOUNT RATE)
 GOVERNMENT DISCOUNT RATE CAN BE EQUAL TO
 THE RATE USED IN PRIVATE SECTORS.

- SOCIAL DISCOUNT RATE REFLECTS A SOCIETY'S RELATIVE VALUATION ON TODAY'S WELL-BEING VERSUS WELL-BEING IN THE FUTURE.
- SO, CHOOSING AN APPROPRIATE SDR IS CRUCIAL FOR CBA.

- IF SETTING SDR TOO HIGH, MANY SOCIALLY DESIRABLE PROJECTS ARE LIKELY TO BE EXCLUDED.
- IF SETTING SDR TOO LOW, YOU ENCOUNTER THE RISKS OF IMPLEMENTING ECONOMICALLY INEFFICIENT PROJECTS.

NOTE : WHEN YOU SET A RELATIVELY HIGH SDR, YOU ATTACH LESS WEIGHT TO BENEFIT & COST STREAMS THAT OCCUR IN THE Distant FUTURE BUT FAVOR PROJECTS W/ BENEFITS & COST OCCURRING AT LATER DATES.

WHEN YOU SET A RELATIVELY LOW SDR,
YOU ARE LIKELY TO FAVOR PROJECTS THAT
BENEFITS OCCURRING AT LATER DATES.

Discount Rate for Government Projects

- Less consensus on appropriate discount rate in public sector. One possibility are rates based on returns in private sector.
 - Assumes all of the money that is raised would have been invested in a private sector project.
 - In reality, funding comes from a variety of sources - investment and consumption.
 - Funding that come from consumption should be discounted at the *after-tax* discount rate.
 - Hard in reality to determine what proportions of funding come from consumption or investment.

⇒ USED WHEN
NO MARKET
DISTURBANCES;
NO EXTERNALITY;

EX

1000 MILLION BAIT

TAXES

BORROWING MONEY FROM PRIVATE SECTOR.

CREATES DISTURBANCES OF CITIZEN'S CHOICES ON CONSUMPTION & SAVING ON LEISURE & WORKING, ETC.

Q: WHAT IS THE MARGINAL SOCIAL OPPORTUNITY COST OF THIS MONEY?

A: IF PRIVATE SECTOR INVESTS THIS AMOUNT AND GET RETURN OF 12%, SO RETURN = 120 MILLION BAIT.

THEN MARGINAL SOCIAL OPPORTUNITY COST OF GOVT. BORROWING = 12% OR 120 MILLION BAIT.

WILL BE AN APPROPRIATE DISCOUNT RATE.

Discount Rate for Government Projects

- Another possibility is the **social rate of discount** – which measures the valuation society place on consumption that is sacrificed in the present.
- Differs from market returns because it:
 - Accounts for *concern about future generations*
 - Involves paternalism
 - May solve some market inefficiency such as positive externalities

GOVT MIGHT USE LOWER DISCOUNT RATE THAN THE RATE USED²⁶ IN PRIVATE SECTORS AS THE PROJECT ITSELF MIGHT HELP SOLVING, LET'S SAY, EXTERNALITY PROBLEM.
EX: WATER TREATMENT PROJECT,
PROJECTS THAT TACKLE GLOBAL WARMING.

Discount Rate for Government Projects

- In reality, federal agencies are required to use a real rate of return of 7%, on the assumption that this measures the before-tax rate of return in the private sector.
- Some use 2% real return instead, thought to measure the after-tax rate of return.

1000 MILLION BAIT
r = 12%, B/F TAX
TAX RATE = 50%
AFTER TAX RATE OR
RETURN = 6%

Discount Rate for Government Projects

- When a new tax or expenditure is introduced, its effects over a 5-year or 10-year period are analyzed to see whether it will put the budget out of balance.
 - Ignores discounting.
 - Costs (or benefits) outside of the window are not counted toward deficit (or surplus).

Valuing Public Benefits and Costs

TANGIBLE INTANGIBLE TANGIBLE
INTANGIBLES

- Recall that the discount rate, benefits, and costs are needed to compute the present value of a project.
- For private company:
 - Benefits = revenues received
 - Costs = firm's payments for inputs

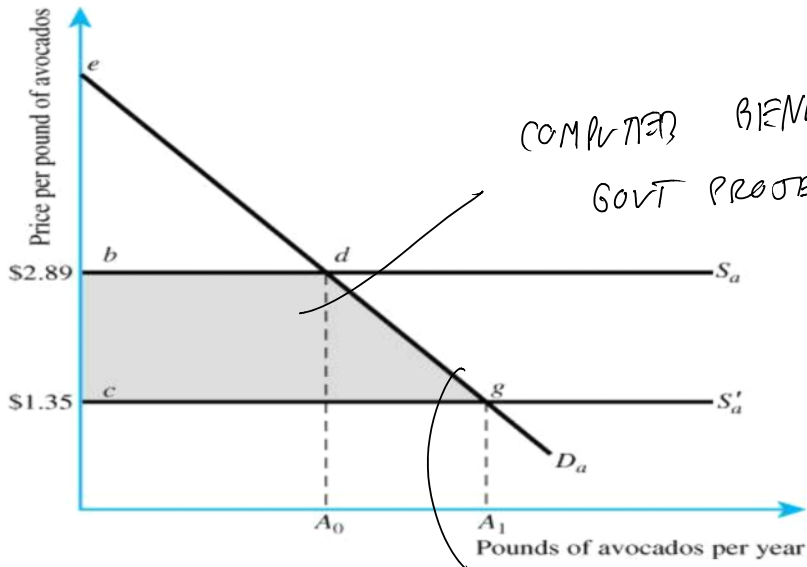
Valuing Public Benefits and Costs

- Consumer surplus
 - Public sector projects can be large and change market prices.
 - Figure 11.1 measures the change in consumer surplus from a government irrigation project that lowers the cost of agricultural production.

Valuing Public Benefits and Costs

- For public sector, market prices may not reflect *social* benefits and costs.
 - Externalities, for example
- Several ways of measuring benefits and costs
 - Market prices
 - Adjusted market prices
 - Consumer surplus
 - Inferences from economic behavior
 - Valuing intangibles

Figure 11.1



COMPUTED BENEFITS OF
GOVT PROTECTION ON AVOCADO
MARKET = $b d g - c$

DEMAND MUST BE ESTIMATED ACCURATELY,
CURVE

Valuing Public Benefits and Costs

- Market prices

- In a properly functioning competitive economy, the price of a good simultaneously reflects its marginal social cost of production and its marginal value to consumers.
- Ignores market imperfections
- Easy to gather

IF MARKET IS COMPETITIVE,

PRICE WILL

REFLECT

MARGINAL COST

OF INPUTS/RESOURCES

USED.

Valuing Public Benefits and Costs

- Adjusted market prices
 - If markets are imperfect, prices generally do not reflect true marginal social cost.
 - **Shadow price** of a commodity is its true, underlying marginal social cost, which can sometimes be estimated.
 - Examples where insights can be gleaned include monopoly price, taxes, and unemployment.

Valuing Public Benefits and Costs

- In this figure, the *change* in consumer surplus is area *bcd*.
- Provided the government planner can accurately measure the demand curve, the project's benefit can be measured with this change.

Valuing Public Benefits and Costs

- Inferences from Economic Behavior
- Many times a good in question is not explicitly traded, so no market price exists.
- Examples:
 - Value of time
 - Value of life

Valuing Public Benefits and Costs

- Value of time
- In cost-benefit analysis, need to estimate the value of time to take advantage of theory of leisure-income choice.
 - After-tax wage is often used
 - But hours of work not always a “choice,” and not all uses of time away from job equivalent.

Valuing Public Benefits and Costs

- Researchers have examined value of time by travel commuting choices.
 - Trains are more expensive, but less time-consuming, than buses. The same is true about non-stop airline flights versus those with a layover.
 - Estimates are that value of time approximately half of the before-tax wage.

Valuing Public Benefits and Costs

- Value of life
- The mindset that “life is priceless” presents obvious difficulties for cost-benefit analysis.
- If the benefits of a saved life are infinite, any project that leads to even a single life saved has an infinitely high present value.

Valuing Public Benefits and Costs

- Economists use two methods to assign finite values to human life:
 - **Lost earnings:** Net present value of individual's after-tax earnings over lifetime.
 - Taken literally, no loss for aged, infirm, or severely handicapped
 - **Probability of death:** Most projects affect *probability* of death (e.g., cancer research). People are willing to accept increases in the probability of death for a finite amount of money.

Valuing Public Benefits and Costs

- Examples:
 - Purchasing a more expensive, safer car with a lower probability of death versus a less expensive, less safe car.
 - Occupational choice: Riskier jobs have higher wages, all else equal
 - Willingness to pay for safety devices like smoke alarms.

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Valuing Public Benefits and Costs

- Estimates suggest value of a life between \$4,000,000-\$9,000,000
- Can contrast this versus the cost per life saved:
 - Emergency floor lights on airplanes cost about \$900,000 per life saved
 - Asbestos removal rules cost \$100,000,000 per life saved

Valuing Public Benefits and Costs

- Valuing intangibles
 - National prestige, others
- Can be used to subvert entire cost-benefit analysis
- Could use difference between costs and benefits to make an argument on how large intangibles would have to be to make the project admissible

$$C - B > 0 \Rightarrow$$

(X=) INTANGIBLE

VALUES LIKE

NATION PRESTIGE

EX: ① $C - B = 10 \text{ MILLION US./YR}$

② $C - B = 10 \text{ BILLION US./YR}$

Cost-Benefit "Games"

- Chain-Reaction game

- Include secondary benefits to make a proposal appear more favorable, without also including the secondary costs

- Labor game

- Wages are viewed as benefits rather than costs of the project

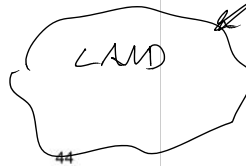
- Double counting game

- Benefits are erroneously counted twice

BENEFITS

COSTS

EX: GOVT
BUILDS
A ROAD



GOVT
CONSIDERS
OF DEVELOPING
THIS LAND

- ① ↑ PRICE OF LAND
- ② NET STREAM OF
INCOME FROM
FARMING

Distributional Considerations

- The ***Hicks-Kaldor criterion*** bases project selection on whether there is a potential Pareto improvement
 - May imposes costs on some if benefits on others are larger
- Others view some groups in population as “more deserving” and argue this should be taken into account in project selection

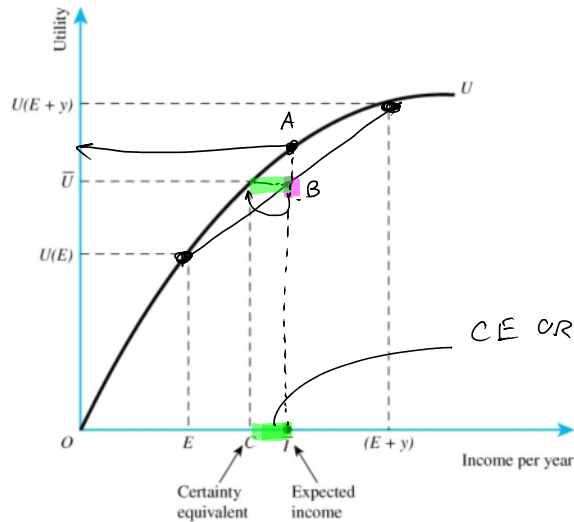
Uncertainty

- The results of many projects are *uncertain* (e.g., AIDS vaccine research or defense research).
- In risky projects, benefits or costs must be converted into ***certainty equivalents*** – the amount of certain income the individual would trade for a set of uncertain outcomes generated by project.

Uncertainty

- Requires information on distribution of returns and risk aversion.
- Figure 11.2 shows a risky project ($E, E+y$) and a certain project (C) that give the same expected utility.

Figure 11.2



2 CHOICES

① NON-RISKY JOB $\rightarrow I = \bar{I}$ (POINT A)

② RISKY JOB (POINT B)

EXPECTED INCOME = \bar{I}

CE OR RISK PREMIUM :

AMOUNT OF
MONEY THAT

AN INDIVIDUAL

IS WILLING TO

PAY TO MOVE HIM

FROM RISKY CHOICE TO

NON-RISKY CHOICE

THAT DELIVERS THE

SAME UTILITY

Uncertainty

- Requires information on distribution of returns and risk aversion.
- Figure 11.2 shows a risky project ($E, E+y$) and a certain project (C) that give the same expected utility.

Recap of Cost-Benefit Analysis

- Present value
- Private sector project evaluation
- Discount rate for government projects
- Valuing public benefits and costs
- Cost-benefit “games”
- Distributional considerations
- Uncertainty