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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

$\bar{Y} = 3.2125$ $\bar{X} = 77.625$

Student	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	2.8	63	-14.625	-0.4125	213.8906	6.0328
2	3.4	72	-5.625	0.1875	31.6406	-1.0547
3	3.0	78	0.375	-0.2125	0.1406	-0.0797
4	3.5	81	3.375	0.2875	11.3906	0.9703
5	3.6	87	9.375	0.3875	87.8906	3.6328
6	3.0	75	-2.625	-0.2125	6.8906	0.5578
7	2.7	75	-2.625	-0.5125	6.8906	1.3453
8	3.7	90	12.375	0.4875	153.1406	6.0328
					$\Sigma = 511.8748$	$\Sigma = 17.4347$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$$\beta_1 = \bar{Y}_i - \beta_2 \bar{X}$$

$$\beta_2 = \frac{\mathrm{cov}(x_i, y_i)}{\mathrm{Var}(x_i)} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} = \frac{17.4347}{511.8748} = 0.0341$$

$$\beta_1 = \bar{Y}_i - \beta_2 \bar{x}$$

$$\beta_1 = 3.2125 - (0.0341)(77.625)$$

$$\beta_1 = 0.5655$$

$$Y_i = 0.5655 + 0.0341\, x_i + u_i$$

$\bar{y} = 3.2125$ $\quad x = 77.625$

Student	Y_i	X_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})(y_i - \bar{y})$	x_i^2
1	2.8	63	-14.625	-0.4125	213.8906	6.0328	3969
2	3.4	72	-5.625	0.1875	31.6406	-1.0547	5184
3	3.0	78	0.375	-0.2125	0.1406	-0.0797	6084
4	3.5	81	3.375	0.2875	11.3906	0.9703	6561
5	3.6	87	9.375	0.3875	87.8906	3.6328	7569
6	3.0	75	-2.625	-0.2125	6.8906	0.5578	5625
7	2.7	75	-2.625	-0.5125	6.8906	1.3453	5625
8	3.7	90	12.375	0.4875	153.1406	6.0328	8100
					$\Sigma = 511.8748$	$\Sigma = 17.4347$	$\Sigma = 48,717$

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.2 $\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 x_i$

$\hat{y}_1 = 0.5655 + 0.0341(63)$
$= 2.7138$

$\hat{y}_2 = 0.5655 + 0.0341(72)$
$= 3.0207$

$\hat{y}_3 = 0.5655 + 0.0341(78)$
$= 3.2253$

$\hat{y}_4 = 0.5655 + 0.0341(81)$
$= 3.3276$

$\hat{y}_5 = 0.5655 + 0.0341(87)$
$= 3.5322$

$\hat{y}_6 = 0.5655 + 0.0341(75)$
$= 3.123$

$\hat{y}_7 = 0.5655 + 0.0341(75)$
$= 3.123$

$\hat{y}_8 = 0.5655 + 0.0341(90)$
$= 3.6345$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$

$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$

$\hat{u}_3 = 3.0 - 3.2253 = -0.2253$

$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$

$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$

$\hat{u}_6 = 3 - 3.123 = -0.123$

$\hat{u}_7 = 2.7 - 3.123 = -0.423$

$\hat{u}_8 = 3.7 - 3.6345 = 0.0655.$

$$\sum_{i=1}^{n=8} \hat{u}_i = -0.0001 \approx 0$$

$(\hat{u}_1)^2 = 0.0074$

$(\hat{u}_2)^2 = 0.1439$

$(\hat{u}_3)^2 = 0.0508$

$(\hat{u}_4)^2 = 0.0026$

$(\hat{u}_5)^2 = 0.0046$

$(\hat{u}_6)^2 = 0.0151$

$(\hat{u}_7)^2 = 0.1789$

$(\hat{u}_8)^2 = 0.0043$

$$\sum_{i=1}^{n=8} (\hat{u}_i)^2 = 0.4347$$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$Var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{\sum_{i=1}^{n=8} (\hat{u}^2)}{n-2} = \frac{0.4347}{6} = 0.0725$$

$$SSR = \sum (y_i - \hat{y})^2$$

$$Var(\hat{\beta}_1) = \frac{\sum (\hat{\sigma}_u^2)(X_i^2)}{n \sum x_i^2} = \frac{48717 \cdot 0.0725}{8\,(511.8748)} = \frac{3529.5467}{4{,}094.9984} = 0.8619$$

$$Var(\hat{\beta}_2) = \frac{\sigma_u^2}{\sum (x_i^2)} = \frac{0.0725}{511.8748} = 0.0001$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i}$, $\boldsymbol{u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$\bar{x} = 20$

$\bar{y} = 9.1$

X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(x_i-\bar{x})(y_i-\bar{y})$	x_i^2
10	0	-10	-9.1	100	91	100
12	2	-8	-7.1	64	56.8	144
14	5	-6	-4.1	36	24.6	196
16	6	-4	-3.1	16	12.4	256
18	7	-2	-2.1	4	4.2	324
22	10	2	0.9	4	1.8	484
24	10	4	0.9	16	3.6	576
26	15	6	5.9	36	35.4	676
28	16	8	6.9	64	55.2	784
30	20	10	10.9	100	109	900
				$\Sigma = 440$	$\Sigma = 394$	$\Sigma = 4440$

2.1 $\beta_1 = \bar{Y} - \beta_2 \bar{X}$

$\beta_2 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{394}{440} = 0.8955$

$\beta_1 = 9.1 - 0.8955(20)$

$= -8.81$

$Y_i = -8.81 + 0.8955 x_i + u_i$

2.2 $\hat{Y}_i = \beta_1 + \beta_2 \cdot X_i$

$\hat{Y}_1 = -8.81 + 0.8955(10)$
$= 0.145$

$\hat{Y}_2 = -8.81 + 0.8955(12)$
$= 1.936$

$\hat{Y}_3 = -8.81 + 0.8955(14)$
$= 3.727$

$\hat{Y}_4 = -8.81 + 0.8955(16)$
$= 5.518$

$\hat{Y}_5 = -8.81 + 0.8955(18)$
$= 7.309$

$\hat{Y}_6 = -8.81 + 0.8955(22)$
$= 10.891$

$\hat{Y}_7 = -8.81 + 0.8955(24)$
$= 12.682$

$\hat{Y}_8 = -8.81 + 0.8955(26)$
$= 14.473$

$\hat{Y}_9 = -8.81 + 0.8955(28)$
$= 16.264$

$\hat{Y}_{10} = -8.81 + 0.8955(30)$
$= 18.055$

$\hat{u}_i = (Y_i - \hat{Y}_i)$

$\hat{u}_1 = 0 - 0.145 = -0.145$

$\hat{u}_2 = 2 - 1.936 = 0.064$

$\hat{u}_3 = 5 - 3.727 = 1.273$

$\hat{u}_4 = 6 - 5.518 = 0.482$

$\hat{u}_5 = 7 - 7.309 = -0.309$

$\hat{u}_6 = 10 - 10.891 = -0.891$

$\hat{u}_7 = 10 - 12.682 = -2.682$

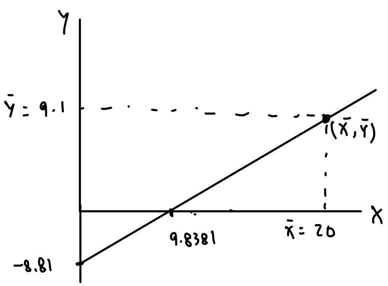
$\hat{u}_8 = 15 - 14.473 = 0.527$

$\hat{u}_9 = 16 - 16.264 = -0.264$

$\hat{u}_{10} = 20 - 18.055 = 1.945.$

$\sum_{i=1}^{n=8} \hat{u}_i \approx 0.$

2.3



$Y_i = -8.81 + 0.8955 X_i + u_i$

$Y_i = -8.81 + 0.8955(20)$
$= 9.1$

$\therefore$ This regression function passes through $(\bar{X}, \bar{Y})$

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

2.4 $Y_i = -8.81 + 0.8955(x_i)$

$Y_i = -8.81 + 0.8955(18)$

$Y_i = 7.309$

2.5 $var(\hat{u}_i) = \dfrac{\sum \hat{u}_i^2}{n-2} = \dfrac{14.0909}{8} = 1.7614$

$Var(\hat{\beta}_1) = \dfrac{\sum (X_i)^2 \cdot \sigma_u^2}{n \sum (x_i - \bar{x})^2} = \dfrac{\cdot 1.7614}{10 \cdot 440} = \dfrac{4440 \cdot 1.7614}{4400} = 1.7614$

$Var(\hat{\beta}_2) = \dfrac{\sigma_u^2}{\sum x_i^2} = \dfrac{1.7614}{440} = 0.004$

$y_i = \beta_1 + \beta_2 X_i + u_i$

$\beta_2 = \dfrac{\beta_1 + u_i - y_i}{-X_i}$

$\bar{y} = \beta_1 + \beta_2 \bar{X}$

3. Consider the below regression function: consider the two-variable model

$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

OLS $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \hat{u}_i$

min $\sum \hat{u}_i^2$ $\hat{u}_i = \hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i$

$\sum \hat{u}_i^2 = \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i)^2$

F.O.C.:

$-2 \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i) = 0$

$\sum \hat{y}_i - \sum \hat{\beta}_1 - \beta_2 \sum \hat{x}_i = 0$

$\sum \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$

$n \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$ uns n.

$\hat{\beta}_1 = \bar{y}_i - \beta_2 \bar{x}$

unbiase when $E(\hat{\beta}_1) = \beta_1$

$\hat{\beta}_1 = \bar{y} - \beta_2 \bar{X}$

$\hat{\beta}_1 = \beta_1 + \beta_2 \bar{X} - \beta_2 \bar{X}$

$\hat{\beta}_1 = \beta_1$

$E(\hat{\beta}_1) = \beta_1$