$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 3 \end{bmatrix}.$$

(b) From U in part (a) we see that every column is a pivot column. The pivot columns from A are a basis for the column space: $(2,2,0)$, $(2,5,3)$, $(1,0,2)$. Since the rank is three, the column space is all of $\mathbf{R}^3$, so another basis would be the standard basis $(1,0,0)$, $(0,1,0)$, $(0,0,1)$. In fact, any three independent vectors in $\mathbf{R}^3$ will do.
(c) The rank is three because there are three pivots.

(2)
$$c_1\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + c_3\begin{bmatrix}0\\0\\1\\1\end{bmatrix} + c_4\begin{bmatrix}0\\1\\0\\1\end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix} = 0$$

Perform elimination process

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \cdots \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\underline{c} \neq 0$

$\therefore$ These vectors are <u>dependent</u>.

& They do not span R^4

~~C~~

(3) Columns 2 & 3 are a basis for Col A

(4) Take any $\underline{u}$ in H – say $\underline{u} = \begin{bmatrix}3\\7\end{bmatrix}$ – and take any $c \neq 1$ – say $c=2$.

Then $c\underline{u} = \begin{bmatrix}6\\14\end{bmatrix}$. If this is in H, then there is some s such that

$\begin{bmatrix}3s\\2+5s\end{bmatrix} = \begin{bmatrix}6\\14\end{bmatrix} \Rightarrow$ That is $s=2$ and $s = \frac{12}{5}$ which is impossible

1/2

solve $A\vec{x} = 0$
Yes. it is in Nul A

(6)

$\underline{w}$ is in Col A $\to$ by checking that $A\underline{x} = \underline{w}$ is consistent
$\underline{w}$ is in Nul A $\to$ by checking $A\underline{x} = 0$
$\to$ vector $\underline{x}$ which is solution to
i.e. $\underline{x} = c\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(7) Let $A = [\underline{v}_1 \ \underline{v}_2]$. Row operation shew that

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 7 \\ 3 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

Not every row of A contains a pivot position. So the columns of A do not span R^3, by Hence $\{\underline{v}_1, \underline{v}_2\}$ is not a basis for R^3. Since $\underline{v}_1$ and $\underline{v}_2$ are not in R^2, they cannot possibly be a basis for R^2. However, since $\underline{v}_1$ and $\underline{v}_2$ are obviously linearly independent, they are a basis for a subspace of R^3, namely, Span $\{\underline{v}_1, \underline{v}_2\}$

(8)

$$\begin{bmatrix} 1 & 6 & 2 & -4 \\ 3 & 2 & -2 & -8 \\ -4 & -1 & 3 & 9 \end{bmatrix} \underset{\text{sequence of row operations}}{\sim} \begin{bmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & -3/2 \\ 0 & 0 & \boxed{1} & 5/2 \end{bmatrix}$$

The Basis for W $= \left\{ \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} \right\}$