8 How the Hypothesis Testing is done in Practice

1. Check the values of $t-statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t-statistics$ are to test $H_0 : \beta_i = 0$

   z-table

   $\Rightarrow$ If the d.f. > 30, then when $t \ > 1.96$, we can reject H_0 with 5% sig level

   $\Rightarrow$ **When** $t \ $ **> 1.96**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When** $t \ $ **< 1.96** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t \ < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $\quad H_0 : \beta_i = 5 \quad$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

sales →

company performance

CEO characteristics

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	——	.112 (.050)	.100 (.049)
profmarg	——	−.0023 (.0022)	−.0022 (.0021)
ceoten	——	——	.0171 (.0055)
comten	——	——	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353

like a simple regression with 1 x.

Multiple Regression Analysis : Further Issues

1 Data scaling on OLS statistics

When we change the unit of measur$\overset{e}{\vee}$ment of a variable, the value of estimators would chance accordingly. For example

$$\widehat{bweght}_g = \widehat{\beta}_0 + \widehat{\beta}_1 cigs + \widehat{\beta}_2 fa\min c,$$

where
$bwght =$ child birth weight, in grams.
$cigs =$ number of cigarettes smoked by the mother while pregnant, per day.
$fa\min c =$ annual family income, in thousands of dollars.

- what if we use bweght in kilograms ??

$$1\ kg = 1{,}000\ g$$

$$\widehat{bweght}_{kg} = \frac{\widehat{bweght}_g}{1000} = \frac{\hat\beta_0}{1000} + \frac{\hat\beta_1}{1000} cigs + \frac{\hat\beta_2}{1000} faminc$$

$$= \hat\alpha_0 + \hat\alpha_1 cigs + \hat\alpha_2 faminc$$

$$\Rightarrow \hat\alpha_0 = \frac{\hat\beta_0}{1000}, \quad \hat\alpha_1 = \frac{\hat\beta_1}{1000}, \quad \hat\alpha_2 = \frac{\hat\beta_2}{1000}$$

- What if we use faminc in USD (instead of 1000 USD)

$$bweght_g = \hat\beta_0 + \hat\beta_1 cigs + \frac{\hat\beta_2}{1000} faminc_{USD}$$

The value of this variable is going to be 1000 times larger than faminc

$$= \hat\beta_0 + \hat\beta_1 cigs + \hat\theta_2 faminc_{USD}$$

$$\Rightarrow \hat\theta_2 = \frac{\hat\beta_2}{1000}$$

in other words $\hat\theta_2$ = impact of 1 USD ↑ in income

$\hat\beta_2$ = ⟼ 1000 USD ↑ in income

- what if we use bweght in kg & income in THB

$$bweght_{kg} = \frac{\hat\beta_0}{1000} + \frac{\hat\beta_1}{1000} cigs + \frac{\left(\frac{\hat\beta_2}{1000}\right)}{30{,}000} faminc_{THB}$$

This value is going to be 30,000 times more than faminc

Note:

$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$

$\frac{d \ln x}{dx} = \frac{1}{x} \longrightarrow \boxed{d\ln(x) = \frac{1}{x}\,dx}$

2 More on functional forms

- Logarithmic Functional Form

usually means natural log

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + u$$

$\Delta y = y_1 - y_2$

$\Delta x_1 = x_{11} - x_{12}$

- $$\beta_1 = \frac{d\log(y)}{d\log(x_1)} = \frac{\frac{1}{y}\,dy}{\frac{1}{x_1}\,dx_1} = \frac{\frac{1}{y}\Delta y}{\frac{1}{x_1}\Delta x_1} = \frac{100 \times \frac{1}{y}\Delta y}{100 \times \frac{1}{x_1}\Delta x_1} = \frac{\%\, y}{\%\, x}$$

with the log y & log x format, the coefficient is going to be the elasticity !

(x, elasticity of y)

price demand (Q)

- $$\beta_2 = \frac{d\log(y)}{dx_2} = \frac{\frac{1}{y}dy}{dx_2} = \frac{\frac{1}{y}\Delta y}{\Delta x_2}$$

⇒ if we want the upper term to be % change, then

$$100\,\beta_2 = \frac{100\,\frac{1}{y}\Delta y}{\Delta x_2}$$

$$100\,\beta_2 = \frac{\%\,\Delta y}{\Delta x_2}$$

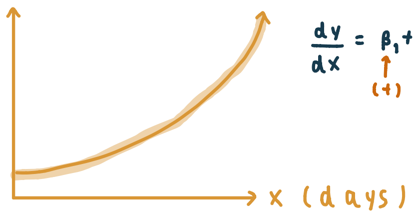
$100\,\beta_2 = \%\,\Delta$ in y given that x_2 increases by 1 unit

- Models with Quadratics (squares)

→ capture increasing / decreasing marginal effects (slope of the relationship between x & y is not constant

COVID-19 example

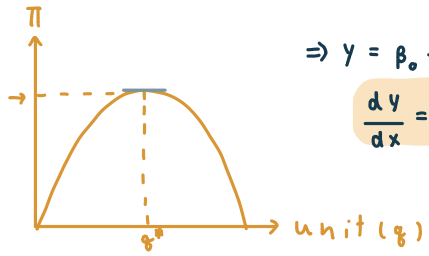
y (# of cases) ⇒ $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$



$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$ (days)

(+) (+)

Decreasing returns



⇒ $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$

(+) (−)

$\pi = (P - MC)\,q$; Assume $MC = 10$

Demand $P = 100 - q$

$\pi = (100 - q - 10)\,q$

F.O.C. $\frac{d\pi}{dq} = 0 = 90 - 2Q$

β_1 is positive

β_2 is (−)

Example : Effects of Pullution on Housing Prices

$$\log(price) = \beta_0 + \beta_1 \log(nox) + \beta_2 \log(dist) + \beta_3 rooms + \beta_4 room^2 + \beta_5 stratio + u$$

where

$price$ = housing price
nox = level of pollution
$dist$ = distance from downtown
$rooms$ = number of rooms
$stratio$ = average student per teacher ratio

In the US or many other countries, students can apply to school in the area without having to take any test. So, the lower st ratio, the better the school.

The estimation result is given by

```
regress  lprice lnox dist rooms rooms_sq stratio
```

Source	SS	df	MS
Model	51.4933152	5	10.298663
Residual	33.0889098	500	.06617782
Total	84.582225	505	.167489554

Number of obs	= 506
F(5, 500)	= 155.62
Prob > F	= 0.0000
R-squared	= 0.6088
Adj R-squared	= 0.6049
Root MSE	= .25725

log(price)

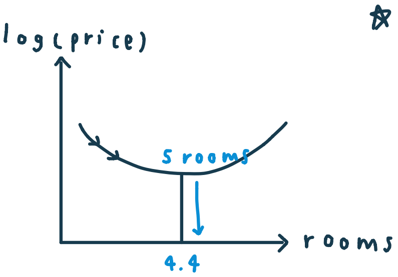
lprice		Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
log(nox) → lnox	β_1	-.9767545	.0995938	-9.81	0.000	-1.172429	-.7810806
dist	β_2	-.0321972	.0094013	-3.42	0.001	-.050668	-.0137264
rooms	β_3	-.5528032	.1612965	-3.43	0.001	-.8697056	-.2359007
rooms_sq	β_4	.0624697	.0124867	5.00	0.000	.0379368	.0870025
stratio	β_5	-.0486667	.0058131	-8.37	0.000	-.0600879	-.0372455
_cons		13.59154	.5650901	24.05	0.000	12.4813	14.70178

$|t| > 1.96$ ↑ ↑ all < 0.05

→ all variables are significant

Consider the effect of "room"

$$\frac{d\log(price)}{d\,rooms} = \beta_3 + 2\beta_4 rooms = -0.553 + 2(0.062)\cdot rooms$$



☆ at how many rooms does 1 additional room has a positive impact on log(price) ??

$$0 = -0.553 + 2(0.062)\cdot rooms$$

$$rooms = 4.4$$

Answer ⇒ at 4.4 rooms or more
at 5 rooms or more

What wold be the % change in price when the number of room increases from 5 to 6?

- $$\frac{d\log(price)}{d\,rooms} = -0.553 + 2(0.062)\cdot rooms$$

$$\frac{100\cdot\frac{1}{price}\,d\,price}{d\,rooms} = 100\,(-0.553 + 2(0.062)\cdot 5)$$

$$= 100 \times 0.067 = \boxed{6.7\,\%} \text{ increase.}$$

→ What about % in price when # rooms increases from 5 to 7 ??

$$\%\Delta\,Price = 100\,(-0.553 + 2(0.062)\cdot 6) = \boxed{19.1\,\%}$$

total % Δ in price when # rooms ↑ from 5 to 7 is 6.7 + 19.1 % = 25.8%

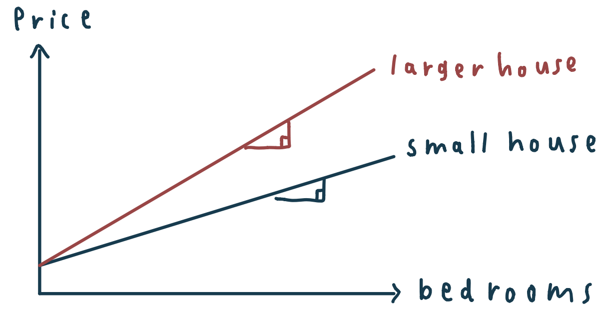
3 Models with Interaction Terms

⇒ used when the impact of one variable depends on the value (level) of another variable.

Consider

$$price = \beta_0 + \beta_1 \underbrace{sqrft}_{x_1} + \beta_2 \underbrace{bdrms}_{x_2} + \beta_3 \overbrace{\underbrace{sqrft}_{x_1} \times \underbrace{bdrms}_{x_2}}^{x_3} + \beta_4 \underbrace{bthrms}_{x_2} + u$$

where
  $price$ = housing price
  $sqrft$ = house size (square feet)
  $bdrms$ = number of bedrooms
  $bthrms$ = number of bathrooms



$$\frac{d\ price}{d\ bdrms} = \beta_2 + \beta_3 sqrft$$

⇒ if $\beta_2 > 0$ then, an additional bedroom would increase price more for a larger house!

4 More on the Goodness-of-Fit and Selection of Regressors

- Adding more regressors ALWAYS improve fit → R^2 always ↑
  - But we lose the "degree of freedom"
    (d.f. = free data point used to estimate the parameter)
    → 1 data point is sacrificed everytime we estimate a parameter.
  - Using R^2 would not punish "having too many regressors"
  - We use adjusted_R^2 or $\bar{R}^2$ when we want to punish adding too many regressors

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{SSR/\cancel{n}}{SST/\cancel{n}}$$

$$adj\ R^2 = \boxed{1 - \frac{SSR/(n-k-1)}{SST/(n-1)}}$$

If we have more k, d.f. $= n-k-1$ ↓, $SSR/(n-k-1)$ ↑, adj- R^2 ↓

Using adjusted R-squared to choose between non-nested models (one model is not a subset of another).

Consider Model 1

$$\begin{aligned} \widehat{salary} &= \underset{(223.90)}{830.63} \ \underset{(0.0089)}{+0.0163 sales} \ \underset{(11.08)}{+19.63 roe} \\ n &= 209, \quad R^2 = 0.029, \quad \bar{R}^2 = 0.020 \end{aligned}$$

Consider Model 2

$$\begin{aligned} \widehat{\log(salary)} &= \underset{(0.29)}{4.36} \ \underset{(0.033)}{+0.2751 \log(sales)} \ \underset{(0.004)}{+0.0179 roe} \\ n &= 209, \quad R^2 = 0.282, \quad \bar{R}^2 = 0.275 \end{aligned}$$

27.5% of variation in y is explained
So, this model is better!

Multiple Regression Analysis with Qualitative Information:

ex. gender, season, month

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories

- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$
$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	*wage*	*educ*	*exper*	*female*	*married*
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
. . .	. . .	. . .	. . .	. . .	. . .
525	11.56	16	5	0	1
526	3.50	14	5	1	0

3 Models with a single dummy independent variable

Consider

$$wage = \beta_0 + \delta_0 \underline{female} + \beta_1 \underline{educ} + u. \quad (1)$$

where

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

In this case, the δ_0 notation is used to highlight the interpretation of the parameters multiplying dummy variables. In other cases, we can use any notation that is the most convenient.

① $E(wage \mid female, educ) = E(\beta_0 + \delta_0 female + \beta_1 educ + u \mid female, educ)$

$= \beta_0 + \delta_0 female + \beta_1 educ + \underbrace{E(u \mid female, educ)}_{=0}$ (ass MLR 1-4 holds)

$= \beta_0 + \delta_0 female + \beta_1 educ$

② Thus,

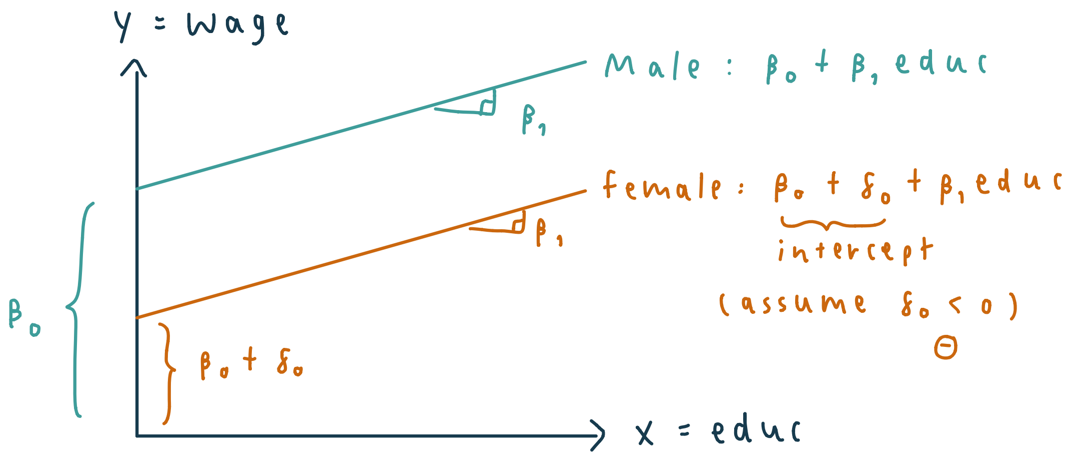
♀ : $E(wage \mid female = 1, educ) = \beta_0 + \delta_0 (1) + \beta_1 educ = \beta_0 + \delta_0 + \beta_1 educ$

♂ : $E(wage \mid male = 0, educ) = \beta_0 + \delta_0 (0) + \beta_1 educ = \beta_0 + \beta_1 educ$

$\delta_0 = E(wage \mid female = 1, educ) - E(wage \mid female = 0, educ)$

or $\delta_0 = E(wage \mid female, educ) - E(wage \mid male, educ)$

* given the same value of educ (same education level)
δ_0 is the difference in the expected wage of females and males.



→ By the way we model this regression function, female is going to give a constant impact on wage, regardless of the level of education.

4 It is not possible to include all of the dummy alternatives in the same model (as long as there is an intercept in the model)

- If we include all alternatives of a dummy variable in the same model, we will face the "perfect collinearity" problem.

ID	female	male	X_0
1	1	0	1
2	1	0	1
3	0	1	1
4	0	1	1
⋮	⋮	⋮	⋮
99			

$$wage = \underset{\uparrow \text{ intercept} \times 1}{\beta_0 X_0} + \delta_0 \underset{(X_1)}{female} + \beta_1 \underset{(X_2)}{educ} + \delta_1 \underset{(X_3)}{male} + u$$

For example:

$$X_0 = X_1 + X_3$$

$$\begin{aligned} 1 &= female + male \\ female &= male + 1 \end{aligned}$$

or

If there are "n" categories, we omit "1" category to avoid multi collinearity

$$\begin{aligned} 1 &= \overset{\checkmark}{winter} + \overset{\checkmark}{spring} + \overset{\checkmark}{summer} + \cancel{fall} \ \checkmark\checkmark \\ winter &= 1 - spring - summer - fall \end{aligned}$$

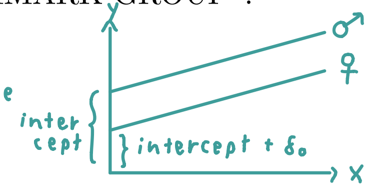
$$winter = \begin{cases} 1 & \text{if winter} \\ 0 & \text{otherwise} \end{cases}$$

$$spring = \begin{cases} 1 & \text{if spring} \\ 0 & \text{otherwise} \end{cases}$$

etc.

ID	winter	spring	summer	fall	X_0
1	1	0	0	0	1
2	1	0	0	0	1
3	0	0	1	0	1
4	0	0		0	1
⋮	0	0	1	0	1
⋮	0	1	0	0	1
⋮	0	1	0	0	1

- At least one alternative has to be dropped. We treat the dropped alternative as the "BASE GROUP" or "BASELINE" or "BENCHMARK GROUP". ← in this case, male



$\begin{cases} 1 & \text{if female} \\ 0 & \text{if male} \end{cases}$ $\begin{cases} 1 & \text{if male} \\ 0 & \text{if female} \end{cases}$

```
. regress lwage female male married educ exper
note: male omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  4,   521) =   75.27
       Model |  54.3265253     4  13.5816313           Prob > F      =  0.0000
    Residual |  94.0032262   521  .180428457           R-squared     =  0.3663
-------------+------------------------------           Adj R-squared =  0.3614
       Total |  148.329751   525   .28253286           Root MSE      =  .42477

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3251146   .0377061    -8.62   0.000    -.3991892    -.25104
        male |          0  (omitted)
     married |   .1380145   .0411197     3.36   0.001     .0572338    .2187953
        educ |   .0872644   .0071554    12.20   0.000     .0732075    .1013213
       exper |   .0076213   .0015314     4.98   0.000     .0046129    .0106297
       _cons |   .4690918   .1040575     4.51   0.000      .264668    .6735156
------------------------------------------------------------------------------
```

Female workers are expected to have less wage compared to male workers.

5 Using dummy variables for multiple categories

Case 1 We can use many dummy variables in the same model

Consider a model which includes 2 dummy variables– *female* and *married*.

female: 1 if female, 0 if otherwise

married: 1 if married, 0 otherwise

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 married + \beta_1 educ + \beta_2 \exp er + \beta_3 \exp er^2 + \beta_4 tenure + \beta_5 tenure^2 + u.$$

```
regress lwage female married educ exper expersq tenure tenursq
```

Source	SS	df	MS
Model	65.6482326	7	9.37831895
Residual	82.6815188	518	.159616832
Total	148.329751	525	.28253286

Number of obs =	526
F(7, 518) =	58.76
Prob > F =	0.0000
R-squared =	0.4426
Adj R-squared =	0.4351
Root MSE =	.39952

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-.2901838	.0361121	-8.04	0.000	-.3611279	-.2192396
married	.0529219	.0407561	1.30	0.195	-.0271456	.1329894
educ	.0791547	.0068003	11.64	0.000	.0657952	.0925143
exper	.0269535	.0053258	5.06	0.000	.0164907	.0374163
expersq	-.0005399	.0001122	-4.81	0.000	-.0007603	-.0003196
tenure	.0312962	.0068482	4.57	0.000	.0178426	.0447499
tenursq	-.0005744	.0002347	-2.45	0.015	-.0010355	-.0001134
_cons	.4177837	.0988662	4.23	0.000	.2235557	.6120116

$\hat{\beta}$

Comments:

1) δ_0 measures the expected difference between female & male workers given the same marital status and other factors.

$$\frac{d \log(wage)}{d\, female} = \frac{\frac{1}{wage}\, d\, wage}{d\, female} = -0.29$$

• female workers are expected to earn less than workers by 29.02% holding other factors the same.

$$\frac{100 \cdot \frac{1}{wage}\, d\, wage}{d\, female} = 100 \cdot (-0.29)$$

$$\frac{\%\Delta wage}{d\, female} = 29.02\%$$

2) δ_1 measures the impact of be married (marriage premium)

But since $|t| < 1.96$ or $p > 0.05$, we do not reject H_0 (Null hypothesis) of no impact.

earn more wage b/c more responsibity

married doesn't has impact on wage.

	♀	♂
marr	marrfem	marrmale
sing	singfem	sing male

Base case

Consider a model which includes dummy variables for each gender/marital status combination– *marrmale*, *marrfem* and *singfem*. (or singmale ← used as the base case)

$$\log(wage) = \beta_0 + \delta_0 marrmale + \delta_1 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u. \quad (8.1)$$

```
regress lwage marrmale marrfem singfem educ exper expersq tenure tenursq

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  8,   517) =   55.25
       Model |  68.3617623     8  8.54522029           Prob > F      =  0.0000
    Residual |  79.9679891   517  .154676961           R-squared     =  0.4609
-------------+------------------------------           Adj R-squared =  0.4525
       Total |  148.329751   525   .28253286           Root MSE      =  .39329

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    marrmale |   .2126757   .0553572     3.84   0.000      .103923    .3214284
     marrfem |  -.1982676   .0578355    -3.43   0.001     -.311889   -.0846462
     singfem |  -.1103502   .0557421    -1.98   0.048     -.219859   -.0008414
        educ |   .0789103   .0066945    11.79   0.000     .0657585     .092062
       exper |   .0268006   .0052428     5.11   0.000     .0165007    .0371005
     expersq |  -.0005352   .0001104    -4.85   0.000    -.0007522   -.0003183
      tenure |   .0290875    .006762     4.30   0.000     .0158031    .0423719
     tenursq |  -.0005331   .0002312    -2.31   0.022    -.0009874   -.0000789
       _cons |   .3213781    .100009     3.21   0.001     .1249041    .5178521
------------------------------------------------------------------------------
```

δ_0, δ_1, δ_2 (marrmale, marrfem, singfem)

$\hat{\beta}$ (educ, exper, expersq, tenure, tenursq)

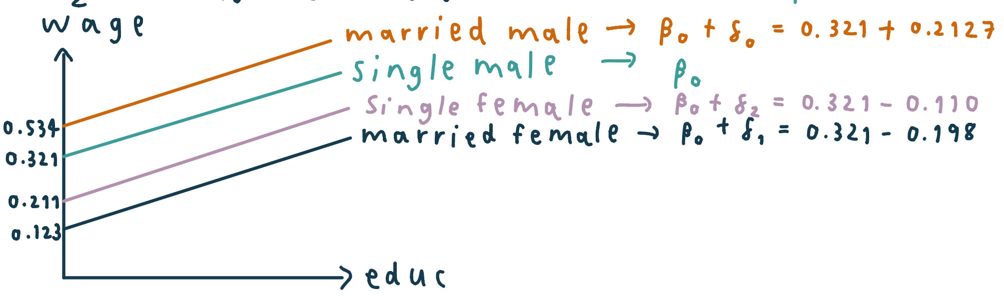
Comments:

This regression is not the same as the previous one. It uses " single Male " as the base group. (The previous one use male & single as 2 base groups).

- δ_0 measure the expected diff in wage of married male as compared with single males, holding other factors constant.

- δ_0 measures the expected diff in ______ married female as compared with single males, holding ______.

- δ_2 → same rationale



Case 2 We can use dummy variables to represent multiple categories of a variable
Consider the relationship between law school rankings and starting salaries

$$\begin{aligned}\log(salary) &= \beta_0 + \delta_0 top10 + \delta_1 r11\_25 + \delta_3 r26\_40 + \delta_4 r41\_60 + \beta_1 LSAT \\ &\quad + \beta_2 GPA + \beta_3 \log(libvol) + \beta_4 \log(cost) + u.\end{aligned}$$

where $top10$, $r11\_25$, $r26\_40$, $r41\_60$ would be equal to 1 when the variable $rank$ falls into the appropriate range.
** Rank below 60 would be the base case.

* In many cases the "range of value" serve as a better explanatory variable than the "value" itself. e.g. age may explain the model better if split into generations young 0-15 gen z 16-29, etc.

```
. regress lsalary top10 r11_25 r26_40 r41_60 LSAT GPA llibvol lcost
```

Source	SS	df	MS
Model	9.16538532	8	1.14567316
Residual	1.2109665	127	.009535169
Total	10.3763518	135	.076861865

Number of obs =	136
F(8, 127) =	120.15
Prob > F =	0.0000
R-squared =	0.8833
Adj R-squared =	0.8759
Root MSE =	.09765

lsalary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
top10	.5393428	.053542	10.07	0.000	.4333927	.6452928
r11_25	.4716199	.0390921	12.06	0.000	.3942637	.548976
r26_40	.2790977	.0346972	8.04	0.000	.2104383	.3477571
r41_60	.182382	.0283098	6.44	0.000	.126362	.238402
LSAT	.0060482	.0034919	1.73	0.086	-.0008616	.012958
GPA	.1305893	.0818678	1.60	0.113	-.0314122	.2925908
llibvol	.0725522	.0289213	2.51	0.013	.0153221	.1297824
lcost	.0249169	.0283224	0.88	0.381	-.031128	.0809619
_cons	8.363103	.4457314	18.76	0.000	7.481081	9.245125

the base line is ranking 61th and worse

Comments:

rank	top 10	r 11 - 25	r26 - 40
1	1	0	0
2	1	0	0
3	1	0	0
⋮	1	⋮	0
10	1	0	0
11	0	1	0
12	0	1	0
⋮	0	1	1
25	0	1	1
26	0	0	1
⋮	0	0	1
40	0	0	1
⋮	0	0	

etc.

1) δ_0 measures the difference in expected log(salary) of a law-school graduate from a top 10 university compared to expected log(salary) of those who graduated from the school ranked 61th and worse.

2) δ_1 → use the same rationale.