B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 2/2013

Practice Problem 4 (Matrix Algebra and Applications)[1][2]

Suggested Answers

1. Write the following systems as matrix equations:

a) $x_1 + 2x_2 + x_3 = 4$

$x_1 - x_2 + x_3 = 5$

$x_1 - x_2 + 2x_3 = 15$

b) $2x_1 - 3x_2 + x_3 = 0$

$x_1 + x_2 - x_3 = 0$

2. For what values of a is $\begin{bmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{bmatrix}$ symmetric?

3. Find the following determinants.

a) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

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[1] Questions with asterisks (*) in the front are "harder" problems.

[2] Questions 2-9 are from Sydsaeter and Hammond (2008), and questions 10-11 are from Dowling (2012)

c) $\begin{vmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{vmatrix}$

d) $\begin{vmatrix} a & 0 & b \\ 0 & e & 0 \\ c & 0 & d \end{vmatrix}$

4. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{bmatrix}$.

Calculate AB, |A|, |B|, |A|·|B| and |AB|.

5. Prove that the inverse of $\begin{bmatrix} 1 & 1 & -3 \\ 2 & 1 & -3 \\ 2 & 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 1 & 0 \\ 8/7 & -1 & 3/7 \\ -2/7 & 0 & 1/7 \end{bmatrix}$.

6. Use Cramer's rule to solve the following systems of equations. Check your answers.

a)
$$\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ x_1 + x_2 - x_3 &= 0 \\ -x_1 - x_2 - x_3 &= -6 \end{aligned}$$

b)
$$\begin{aligned} x_1 - x_2 &= 0 \\ x_1 + 3x_2 + 2x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

7. Consider the simple macro model described by the three equations

(i) $Y = C + A_0$ (ii) $C = a + b(Y-T)$ (iii) $T = d + tY$

where Y is income, C is consumption, T is tax revenue, A_0 is the constant (exogenous) autonomous expenditure, and a, b, d, and t are all positive parameters. Find the equilibrium values of the endogenous variables Y, C, and T by writing the equations in matrix form and applying Cramer's rule.

*8. Examine for what values of the constants a and b the system of equations

$ax + y = 3$
$x + z = 2$
$y + az + bu = 6$
$y + u = 1$

has a unique solution in the unknown $x, y, z,$ and u. Find the unique solution (expressed in terms of a and b).

*9. Consider and economy divided into an agricultural sector (A) and an industrial sector (I). To produce one unit in sector A requires 1/6 unit from A and 1/4 unit from I. To produce in sector I requires 1/4 unit from A and 1/4 unit from I. Suppose final demands in each of the two sectors are 60 units.

a) Write down the Leontif system for this economy.
b) Find the number of units that has to be produced in each sector in order to meet the final demands.

10. Use matrix inversion to solve for the unknowns in the system of linear equations given below.

$$2x_1 + 4x_2 - 3x_3 = 12$$

$$3x_1 - 5x_2 + 2x_3 = 13$$

$$-x_1 + 3x_2 + 2x_3 = 17$$

11. Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.

12. Find the inverse of the following matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 2 & 3 \\ 3 & 0 & 2 \end{bmatrix}.$$

*13. Given the following supply and demand functions:

$$Q_D = 100 - 3P$$

$$Q_S = 80 + 2P$$

a) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

b) Use <u>matrix inversion</u> to solve for the equilibrium quantity and equilibrium price.

c) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Use <u>Cramer's rule</u> to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

14. Consider the following IS-LM model:

Commodity market:

$Y = C + I + G_0 + X_0 - M$

$C = bY_d, \qquad (0 < b < 1)$

$Y_d = Y - T, \quad T \text{ is a constant}.$

$M = mY, \qquad (0 < m < 1)$

$I = I_0 - ar, \quad (I_0 > 0, a > 0)$

Money market:

$M_S = M_0$

$M_D = mY - hr, \quad (m > 0, h > 0)$

a) Write out the explicit IS-LM system of equations, and write a matrix form of these IS-LM equations.

b) Solve for the equilibrium level of national income and rate of interest by using Cramer's rule.