Table of Contents

Chapter 1: Nature of economics analysis and Element of mathematical economics model

1.1 Structure of economic analysis

- How do we study economics? → *Model-based framework*

- What is a model?
    - Model is an *abstract or a simplifed* form of the problem that we are interested in, e.g. anatomy model, map, and etc.
    - The problem that we (economists) are interested in aims at getting us some insights, *helping us understand something about how the world/economy works.*
        - Behavior of agents!
        - Interaction among agents that generate the equilibrium outcome!
    - Typically, all the results in a certain model hinges on the *assumptions* used in the analysis.

- Purpose of economics model; they are all developed to answer three broad classes of analytical questions in economics;

    - *Static Equilibrium (Behavioral) Analysis – at what price do we trade? // Behavior of firms? // Behavior of consumers?*
    - *Comparative Static Equilibrium (Behavioral) analysis – what happen to the trading price when something happen? // What happen to firms*
    - *Dynamic Equilibrium (Behavioral) Analysis – what is the adjustment process toward the new trading price? How do firms set price over time?*

Example 1.A.1: Behavior of a competitive firm

Firm owns a **production technology**. → Economist aims to provide some insight about the behavior of a firm that operates under **a perfectly competitive environment**.

Assumption: Firm's maximize the profit. (Does it make sense?) With a competitive enrionment, the firm takes the market price as given.

Given the maximizing principle, firms choose for the **appropriate level of output and input** so that the profit is maximized.

Behavioral analysis aims to understand about the determination *(Static behavioral analysis)* of *"optimal output" (output supply)* and *"optimal input" (input demand)*, and seeks to perform the *sensitivity analysis (Comparative beahvioral analysis)*, i.e. checking what happen if the determinants change.

As we learn from the first year, the first principle suggests that firms choose for an optimal level of output where P = MC, and increase the amount of production when price increases.

- Behavioral foundataion of "individual" output supply cuve under a perfectly competitive market.

Example 1.A.2: ***The Basic 101 Model, The market mechanism model!***

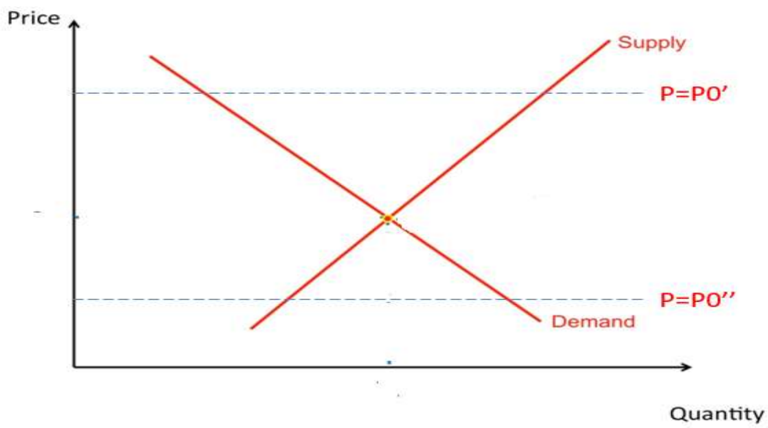
Question: At what price and quantity of output, will the *buyers and sellers* trade in the market?

- To understand the trading outcome, economists model the interaction of the two trading parties in the market using the basic **demand and supply model**.
  - The model basically builds upon several assumptions.

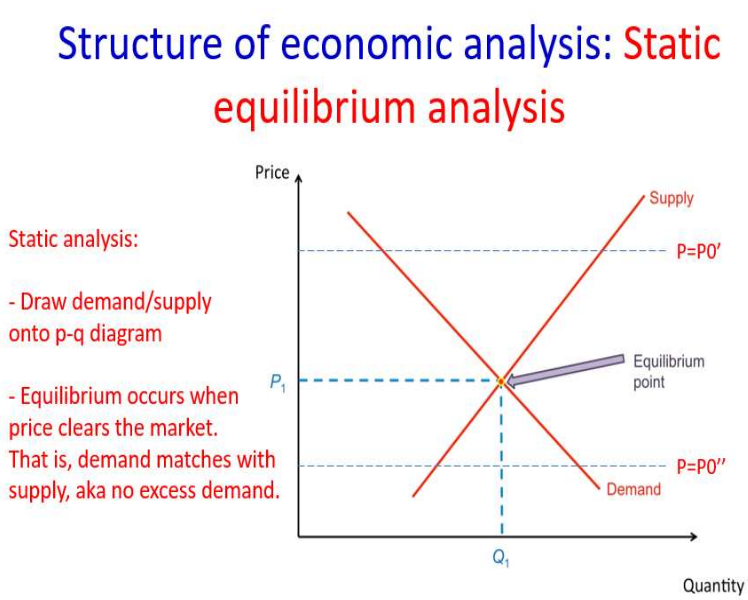
- Conventionally, two behavioral *assumptions* are that
  - **Demand**: Given *all other factors*, quantity demanded is negatively related to price
    - *Downward sloping curve*
  - **Supply**: Given *all other factors*, quantity supplied is positively related to price.
    - *Upward sloping curve*

- Some other **implicit assumptions** include (i) *agents are allowed to adjust their price freely*, (ii) *trading can repeatedly occur over times under a decentralized system.*

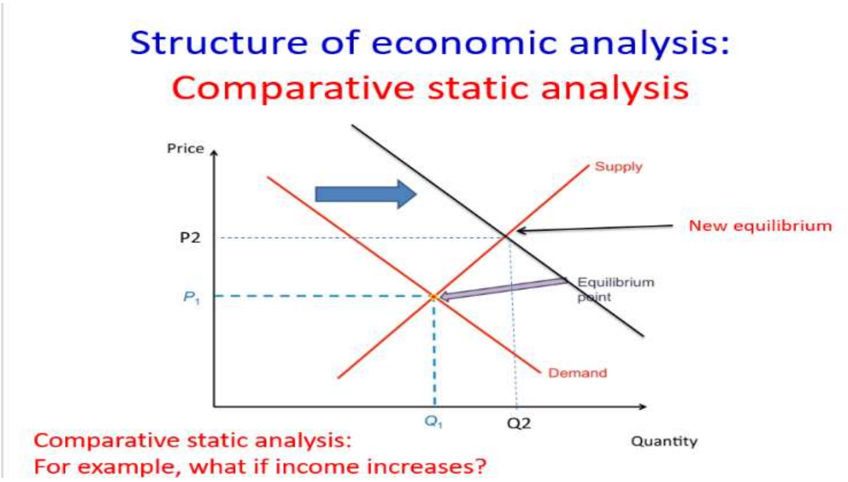
To see how they interact and the way outcome is determined, the model is mostly represented with the aid of diagram.



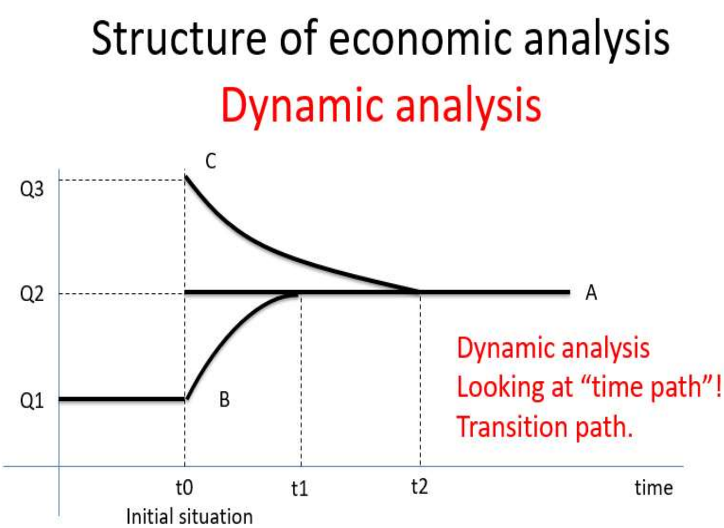
- If P = P0' and P = P0'', what are the trading outcome under each respective cases?
  - Obviously, through the repeated interaction, neither prices represent the steady outcome emerged under repeatedly trading environment.



- Economists aim to uncover the equilibrium! The notion of equilibrium is then defined as a *sustained* outcome with *no further adjustments*: *Static equilibrium analysis*
  - With the aid of diagram, the equilibrium is illustrated by the intersection between "demand" and "supply" curve.



- Economists typically ask further about what would happen to the equilibrium if all those presumed existing conditions turn out to be varying: *comparative static analysis.*



- In many cases, economists are interested in seeing the adjustment process over time, i.e. transitional process. This issue is related to "*dynamic analysis*".

Roadmap for this semester

Where we are headed in this semester is to formalize all these analytical questions by using *mathematical tools.* → Rewrite and Analyze the model concepts discussed in the first year using Mathematics!

There are at least two (important) reasons why we reformulate the whole economic concepts/problems/questions in terms of mathematics.

- First, mathematics is more general approach to represent ideas. Therefore, this thus allows us to easily address/conceptualize the problem in *higher/multi dimensional analysis*.

- Second, economists need to deal with empirical analysis, i. e. validate the model, and *cross-check the model implications with data.*
  - While qualitative predictions are important, many are more interested in quantitative ones.
  - This fundamentally requires the formulation of a model in its quantitative version; mathematics evidently come into play.
  - In addition, statistical techniques can be linked to mathematical models, allowing for a more rigorous validation of model against the data.

1.2 Economics model in Math framework

Formulating economic concepts in terms of math requires the representation.

- Variables
  - Quantity or amount of some objects whose value can be changed.
  - For example, Y is a variable that represents "your income in a day". X is another variable that represents how many hours you have worked in a day. (The value of both Y and X can be changed.)

- Equation
  - An equation describes the relationship of variables, at the least two.
  - How Y and X can be linked together is summarized by an equation.
  - For example, $Y = 9X$ if hourly wage (w) is \$9.

In general, economics usually discusses about the *interrelationship/interlinkage* among variables, and how the interaction brings about the so called *"behavrioal outcome"* or "*equilibrium*". Therefore, most economics model are best represented by a class of mathematical system so called "*system of simultaneous equation(s)*". The system of simultaneous equations takes the following representation,

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m.
\end{aligned}
$$

The example above is a system of (simultaneous) equations where there are M linear equations and N (economic) variables. Each M equation describes the interlink among each of the N variables in a ***linear*** fashion; the model is then called ***linear system***. Note that the model can be set up and take into account some features of ***non-linearity***, in which each equation can be expressed in terms of ***non-linear form***. That class of model will then label as the ***non-linear system***.

To understand the common structure and basic terminologies (jargons) used in mathematical economics, consider "*Example 1.B.1*" which is the reformulation of the basic market equilibrium model using mathematical expression, and "*Example 1.B.2*" which is the reformulation of the behavioal model of a competitive firm.

Example 1.B.1: Consider the market equilibrium model as an example.

Equation 1: $Q^d = a - b * P - c * Y^d$;

Equation 2: $Y^d \equiv Y - T$

Equation 3: $Q^s = d + e * P - f * Temp$;

Equation 4: $Q^d = Q^s$

Notations defined as follows:

- Q^d = quantity demanded, Q^s = quantity supplied, P = price,
- Y^d = total disposable income, Y = Gross income, T = Lump-sum tax, Temp= temperature ➔ variables
- (a, b, c, d, e, f) = *coefficients* or *parameters* that are all positive

Form the given model, there are *four equations*. There are *a number of variables*. To organize our mathematical thinking in a systematic fashion, let's consider the following issues.

1) **Treatment of the type of equations:**
   - Behavioral equations
   - Equilibrium conditions
   - Definitional equation
2) **Treatment of the variables:**
   - Single equation approach:
     - Independent variable (*Explanatory*)
     - Dependent variable
   - System-based approach
     - Endogenous variable:__________________________
     - Exogenous variable:____________________________

Example 1.B.2: Behavior of a competitive firm

A competitive firm owns a production function, which could be alternatively represented by the cost function. It makes choice of decision on Q, taking price (P) as given.

$\pi \equiv TR - TC$ (1)

$TR = P * Q$ (2)

$TC = C(Q)$ (3)

$\pi(Q) = P * Q - C(Q)$ (4)

The **profit function** given by (4) depends on "Q".

While Q is the explanatory variable in the equation (4), Q is not exogenous variable in the written model.

Recall that the goal of this model is to solve for "Q" that maximizies the profit. **So, Q is endogenously determined along the profit-maximizing rule! (We will bring in the calculus to solve for this first-principle problem.)**

Firms make choice of optimal Q by treating "P" as given, and hence treating P as an **exogenous variable!.**

1.2.1) Static Equilibrium analysis in Math framework!

From the two examples above 1.B.1 and 1.B.2, the most common goal of studying the model is to study the equilibrium analysis, i.e. to solve for the solution of the written model. As the exogenous variables are treated as given, solution to the setp-up model (single equation or system of equation) would represent the *endogenous equilibrium (behavioral) solution*. Technically speaking, endogenous equilibrium solution is called the *reduced-form equation*.

Reduced-form equation is the mathematical function that describes the behavior of the endogenous equilibrium solution. *Typically, the function would depend on (i) exogenous variables and (ii) parameters.*

- Type of variables treated/distinguished in the model
  - **Exo**genous variables = treated as "***given***" in the model. Their values are given as known thing, and *will not be solved* within the model.
  - **Endo**genous variable = set of variables that will be solved for.

Example 1.C.1: National income model

A very basic/simple version of Keynesian cross model

- Consumption function: $C = \mathrm{C}(\mathrm{Y_d}, \mathrm{a}, \mathrm{b})$
- Disposable income: $\mathrm{Y_d} \equiv Y - T$
- Tax equation: $T = T_0$
- Investment equation: $I = I_0$
- Government equation: $G = G_0$
- Equilibrium condition: $Y = C + I + G$

Variable Lists:

Y = income, C = consumption, $\mathrm{Y_d}$ =Disposable income, $T =$ tax
I = investment and G = government expenditure

- ***Static equilibrium analysis***: characterizing the equilibrium solution of the model by solving for a solution of the system of equations. By the definition, the reduced-form solution takes the following *general-form representation.*

$$Y^* = f(I_0, G_0, T_0, a, b)$$

$$C^* = g(I_0, G_0, T_0, a, b)$$

$$Y_d^* = h(I_0, G_0, T_0, a, b)$$

- If we assume the function form, the reduced-from solution would have an analytical (specific) solution which is specific to the assumption on the functional form of behavioral equation.
    - For example, we might assume that $\mathrm{C}(\mathrm{Y}, \mathrm{a}, \mathrm{b}) = \mathrm{a} + \mathrm{bY_d}$

Keywords: *Endogenous equilibrium solution, Reduced-form equations, general-form solution, specific-form solution*

1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

- Numerically, if a = 1, T_0 = \$0 I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

Question: What if G is now changed to \$2, how big is the change in Y^* and C^*?

Answer:

New Y^* = ____________________ ➔

A \$1 increase in G causes an increase in Y^* by ____________

In the later part, we will try to figure out the value of *multiplier* when we don't assign any numerical values of exogenous variables.

- The idea is simple. *We just apply the derivative method to the equilibrium solution function.*
- That is, we calculate the value of $\frac{\partial Y^*}{\partial I_0}, \frac{\partial Y^*}{\partial G_0}, \frac{\partial Y^*}{\partial a}, \frac{\partial Y^*}{\partial b}$.

Example 1.C.2: Behavior of a competitive firm

A competitive firm owns a production function, which could be alternatively represented by the cost function. It makes choice of decision on Q, taking price (P) as given.

$\pi \equiv TR - TC$ (1)

$TR = P * Q$ (2)

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$\pi(Q) = P * Q - C(Q)$ (4)

To solve the model, we bring in the calcullus. The level of Q that maximize equation (4) can obtained by solving for the solution of the **first-order condition**.

$\frac{d\pi(Q)}{dQ} = 0; \rightarrow P - \frac{dC(Q)}{dQ} = 0 \rightarrow Q^* = f(P) \rightarrow$ optimal behavior function / Endogenous behvaorial solution (function) / Economically, this function represents the output supply function.

$\frac{dQ^*}{dP}$ =? ? ? → Sensitivty analysis