Chapter 12
Capturing Surplus

Solutions to Problems

11. Softco is a software company that sells a patented computer program to businesses. Each business it serves has the demand for Softco's product P = 70 – 0.5Q. The marginal cost for each program is \$10. Assume there is no fixed cost.

(a) If Softco sells its program at a uniform price, what price would maximize profit? How many units would it sell to each business customer? How much profit would it earn from each business customer?

(b) Softco would like to know if it is possible to improve its profit by implementing block pricing. Suppose that Softco were to sell the first block at the price you determined in (a), and that the quantity for that block is the quantity you determined in (a). Find the profit-maximizing quantity and price per unit for the second block. How much extra profit would Softco earn from each of its business customers?

(c) Now, reconsider your answer to (b). Find the structure of prices and quantities in each of the two blocks that maximizes profit. In other words, you no longer assume that the price and quantity that you determined in (a) is fixed. Instead, you must find the optimal price for both blocks.

For Questions (a) and (b), you can solve them by setting MC = MR.

(a)
Using the monopoly mid-point rule, P* = (70 + 10) / 2 = 40.
Using the demand curve to solve for Q*, 40 = 70 – 0.5Q, so Q* = 60.

(b)
Softco sold the first 60 units at the price of 40.

The residual demand that is not fulfilled by Softco is the horizontal difference between the market demand and the first 60 units sold.

From P = 70 – 0.5Q, the market demand is Qmkt = 140 – 2P.
The residual demand is Qres = Qmkt – 60 = 80 – 2P.
Rearrange Qres into the inverse demand, we get P = 40 – 0.5Q.

Next, Softco maximizes profit subject to the residual demand.
(Here, you can also solve by setting MR = MC.)
Using the monopoly mid-point rule, P_2* = (40 + 10) / 2 = 25.
Using the residual demand curve to solve for Q_2*, 25 = 40 – 0.5Q, so Q_2* = 30.

In conclusion, Softco sells the first 60 units at P = 40. Then it sells another 30 units at P = 25. The total output sold = 60 + 30 = 90.

(c)
Please see another solution note for this question.

12. Consider a market with 100 identical individuals, each with the demand schedule for electricity of P = 10 - Q. They are served by an electric utility that operates with a fixed cost 1,200 and a constant marginal cost of 2. A regulator would like to introduce a two-part tariff, where S is a fixed subscription charge and m is a usage charge per unit of electricity consumed. How should the regulator set S and m to maximize the sum of consumer and producer surplus while allowing the firm to earn exactly zero economic profit?

To maximize the sum of consumer and producer surplus, the regulator must set the usage charge m = 2; this will induce consumers to buy units of electricity as long as their willingness to pay is at least as high as the marginal cost of providing electricity service. This means that each consumer will buy 8 units of electricity. There will be zero deadweight loss in the market.

If there were no subscription charge, each consumer would realize a consumer surplus of 0.5*(10 – 2)*8 = 32. This means that each consumer will be willing to buy electricity as long as the subscription charge is less than 32.

With 100 consumers, the electric utility can then charge each customer a subscription fee of $12 to cover its fixed costs of $1200, leaving each consumer with a consumer surplus of 32 – 12 = 20. So the total revenue for the firm will be the sum of the revenue from the subscription charge (1200) and the revenue from the usage charge 100(8)(2 – 2) = 0. Total revenue will just cover total cost, and the firm will earn zero economic profit.

12. Consider a bar whose owner plans to set profit-maximizing two-part tariff (entry fee and per-drink price) on two types of customers. The owner would like to welcome both types into his bar, meaning that he will not charge an entry fee that is too high. There are 20 people of the X-type whose individual demand is given by P = 10 – Qx. There are 30 people of the Y-type whose individual demand is given by P = 10 – 2Qy. The MC = AC = $2 per drink. Find the optimal entry fee and per-drink price. Also, calculate the profit the bar can make from these 50 customers.**

Please see another solution note for this question.

14. Suppose that Acme Pharmaceutical Company discovers a drug that cures the common cold. Acme has plants in both the United States and Europe and can manufacture the drug on either continent at a marginal cost of 10. Assume there are no fixed costs. In Europe, the demand for the drug is QE = 70 - PE, where QE is the quantity demanded when the price in Europe is PE. In the United States, the

demand for the drug is QU = 110 - PU, where QU is the quantity demanded when the price in the United States is PU.

a) If the firm can engage in third-degree price discrimination, what price should it set on each continent to maximize its profit?

b) Assume now that it is illegal for the firm to price discriminate, so that it can charge only a single price P on both continents. What price will it charge, and what profits will it earn?

a) With third-degree price discrimination the firm should set $MR = MC$ in each market to determine price and quantity. Thus, in Europe setting $MR = MC$

$$70 - 2Q_E = 10$$
$$Q_E = 30$$

At this quantity, price will be $P_E = 40$. Profit in Europe is then $\pi_E = (P_E - 10)Q_E = (40 - 10)30 = 900$. Setting $MR = MC$ in the US implies

$$110 - 2Q_U = 10$$
$$Q_U = 50$$

At this quantity price will be $P_U = 60$. Profit in the US will then be $\pi_U = (P_U - 10)Q_U = (60 - 10)50 = 2500$. Total profit will be $\pi = 3400$.

b) If the firm can only sell the drug at one price, it will set the price to maximize total profit. The total demand the firm will face is $Q = Q_E + Q_U$. In this case

$$Q = 70 - P + 110 - P$$
$$Q = 180 - 2P$$

The inverse demand is then $P = 90 - 0.5Q$. Since $MC = 10$, setting $MR = MC$ implies

$$90 - Q = 10$$
$$Q = 80$$

At this quantity price will be $P = 50$. If the firm sets price at 50, the firm will sell $Q_E = 20$ and $Q_U = 60$. Profit will be $\pi = 50(80) - 10(80) = 3200$.

20. A cruise line has space for 500 passengers on each voyage. There are two market segments: elderly passengers and younger passengers. The demand curve for the elderly market segment is $Q_1 = 750 - 4P_1$. The demand curve for the younger market segment is $Q_2 = 850 - 2P_2$. In each equation, Q denotes the number of passengers on a cruise of a given length and P denotes the price per day. The marginal cost of serving a passenger of either type is $40 per person per day. Assuming the cruise line can price discriminate, what is the profit-

maximizing number of passengers of each type? What is the profit-maximizing price for each type of passenger?

$Q1 = 750 - 4P1 \Rightarrow P1 = 187.5 - (1/4)Q1$. This implies $MR1 = 187.5 - (1/2)Q1$.
$Q2 = 850 - 2P2 \Rightarrow P2 = 425 - (1/2)Q2$. This implies $MR2 = 425 - Q2$.

When we have limited capacity we solve the following two equations:
$MR1 = MR2 \Rightarrow 187.5 - (1/2)Q1 = 425 - Q2$. (Equate the MRs)
$Q1 + Q2 = 500$. (Quantities sold must add up to capacity)

We thus have two equations in two unknowns:
$187.5 - (1/2)Q1 = 425 - Q2$
$Q1 + Q2 = 500$

Solving these equations yields:
$Q1 = 175$.
$Q2 = 325$.

Plugging these back into the inverse demand curves gives us the profit-maximizing prices:
$P1 = 187.5 - (1/4)(175) = 143.75$.
$P2 = 425 - (1/2)(325) = 262.5$.

22. You are the only European firm selling vacation trips to the North Pole. You know only three customers are in the market. You offer two services, round trip airfare and a stay at the Polar Bear Hotel. It costs you 300 euros to host a traveler at the Polar Bear and 300 euros for the airfare. If you do not bundle the services, a customer might buy your airfare but not stay at the hotel. A customer could also travel to the North Pole in some other way (by private plane), but still stay at the Polar Bear. The customers have the following reservation prices for these services:

Reservation Prices (in euros)

Customer	Airfare	Hotel
1	100	800
2	500	500
3	800	100

a) If you do not bundle the hotel and airfare, what are the optimal prices PA and PH, and what profits do you earn?
b) If you only sell the hotel and airfare in a bundle, what is the optimal price of the bundle PB, and what profits do you earn?
c) If you follow a strategy of mixed bundling, what are the optimal prices of the separate hotel, the separate airfare, and the bundle (PA, PH, and PB, respectively) and what profits do you earn?

a) Without bundling, the best the firm can do is set the price of airfare at $800 and the price of hotel at $800. In each case the firm attracts a single customer and earns profit of $500 from each for a total profit of $1000. The firm could attract two customers for each service at a price of $500, but it would earn profit of $200 on each customer for a total of $800 profit, less profit than the $800 price.

b) With bundling, the best the firm can do is charge a price of $900 for the airfare and hotel. At this price the firm will attract all three customers and earn $300 profit on each for a total profit of $900. The firm could raise its price to $1000, but then it would only attract one customer and total profit would be $400. Notice that with bundling the firm cannot do as well as it could with mixed bundling. This is because while a) the demands are negatively correlated, a key to increasing profit through bundling, b) customer 1 has a willingness-to-pay for airfare below marginal cost and customer 3 has a willingness-to-pay for hotel below marginal cost. The firm should be able to do better with mixed bundling

c) Because customer 1 has a willingness-to-pay for airfare below marginal cost and customer 3 has willingness-to-pay for hotel below marginal cost, the firm can potentially earn greater profits through mixed bundling. In this problem, if the firm charges $800 for airfare only, $800 for hotel only, and $1000 for the bundle, then customer 1 will purchase hotel only, customer 2 will purchase the bundle, and customer 3 will purchase airfare only. This will earn the firm $1400 profit, implying that mixed bundling is the best option in this problem.

c) Now, reconsider your answer to (b). Find the structure of prices and quantities in each of the two blocks that maximizes profit. In other words, you no longer assume that the price and quantity that you determined in (a) is fixed. Instead, you must find the optimal price for both blocks.

$Q_2 = \frac{Q_1 + 120}{2}$ (mid point)

$PS = TR - VC$

$PS = (Q_1 \cdot P_1) + (P_2)(Q_2 - Q_1) - 10Q_2$

Find Q_1^*, Q_2^*, P_1, P_2

$\pi = (Q_1)(70 - \frac{Q_1}{2}) + (70 - \frac{Q_2}{2})(Q_2 - Q_1) - 10Q_2$

$\pi = 70Q_1 - \frac{Q_1^2}{2} + 70Q_2 - 70Q_1 - \frac{Q_2^2}{2} + \frac{Q_1Q_2}{2} - 10Q_2$

$\frac{\partial\pi}{\partial Q_1} = -Q_1 + \frac{1}{2}Q_2$

$\frac{\partial\pi}{\partial Q_1} = 0 \rightarrow 2Q_1 = Q_2$

$\frac{\partial\pi}{\partial Q_2} = 60 - Q_2 + \frac{1}{2}Q_1$

$\frac{\partial\pi}{\partial Q_2} = 0 \rightarrow Q_2 = \frac{1}{2}Q_1 + 60$

Solving

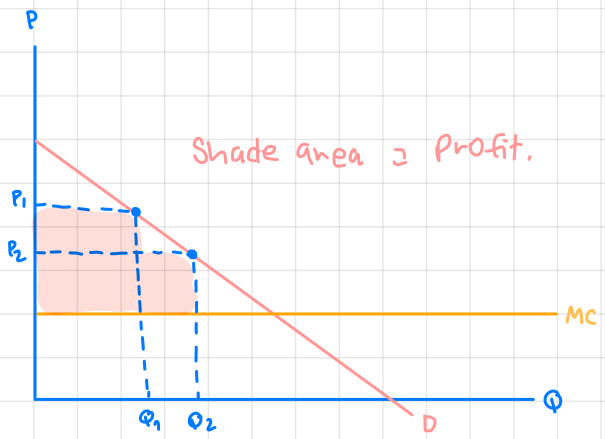
$2Q_1 = \frac{1}{2}Q_1 + 60$

$Q_1 = 40$

$Q_2 = 2Q_1 = 80$

$P_1 = 50$

$P_2 = 30$



12.** Consider a bar whose owner plans to set profit-maximizing two-part tariff (entry fee and per-drink price) on two types of customers. **The owner would like to welcome both types into his bar, meaning that he will not charge an entry fee that is too high.**

There are 20 people of the X-type whose individual demand is given by P = 10 – Qx. There are 30 people of the Y-type whose individual demand is given by P = 10 – 2Qy. The MC = AC = $2 per drink.

Find the optimal entry fee and per-drink price. Also, calculate the profit the bar can make from these 50 customers.

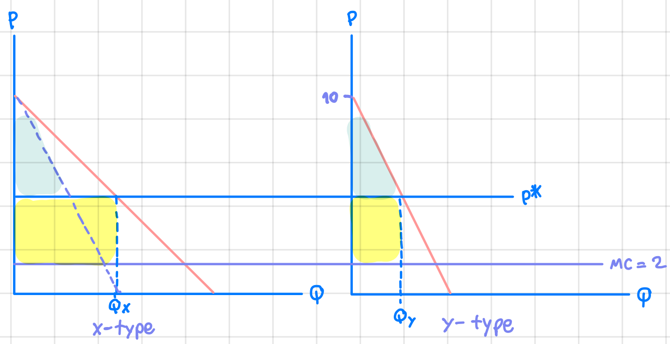
$Q_y = \frac{10-P}{2}$

$Q_x = 10 - P$

X-type
$P = 10 - Q_x$ 20 people

Y-type
$P = 10 - 2Q_y$ 30 people

$MC = AC = \$2$



Total profit = TR − TC

Total Revenue $= 50\left(\frac{1}{2}(10-P)Q_y\right) + 20 Q_x P^* + 30 Q_y P^*$

$= 50\left(25 - 5P + \frac{P^2}{4}\right) + 200P - 20P^2 + 150P - 15P^2$

$= 12.5P^2 - 250P + 1250 + 200P - 20P^2 + 150P - 15P^2$

Total cost $= 20(2 \cdot Q_x) + 30(2 \cdot Q_y)$

$= 40 \cdot (10 - P) + 60 \cdot \left(\frac{10-P}{2}\right)$

$= 400 - 40P + 300 - 30P$

$= 700 - 70P$

Total profit $= 12.5P^2 - 250P + 1250 + 200P - 20P^2 + 150P - 15P^2 - 700 + 70P$

$\frac{d\pi}{dP} = 25P - 250 + 200 - 40P + 150 - 30P + 40 + 30 = 0$

$P^* = 3.78$ — per drink price

Optimal entry fee

$= (10 - 3.78)\left(\frac{10 - 3.78}{2}\right)\frac{1}{2}$

$= 9.67\$$ #

$\pi = (50)\,9.675 + 1.78\,(3.11)(30) + 1.78\,(6.22)(20)$

$= 483.75 + 166.074 + 221.432$

$= 871.256$ #

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$\text{Total } \pi = \pi_{\text{entry fee}} + \pi_{\text{drink sales}}$

$\pi_{\text{entry}} = \Delta \times 50$

$= 50 \times \frac{1}{2}(10 - P)\,(Q_y)$

$= 50 \times \frac{1}{2}(10 - P)\left(\frac{10 - P}{2}\right)$

$= \frac{50}{4}(10 - P)^2$

$\pi_{\text{drink sale}} = 20 \times \square + 30 \times \square$

$= (P - MC)\,Q_x\,(20) + (P - MC)\,Q_y\,(30)$

$= (P - MC)(20\,Q_x + 30\,Q_y)$

$= (P - 2)\left(20(10 - P) + 30\,\frac{(10 - P)}{2}\right)$

$= 35(P - 2)(10 - P)$

$\pi_{\text{total}} = \frac{50}{4}(10 - P)^2 + 35(P - 2)(10 - P)$

Find P that max π $\Rightarrow$ $\frac{\partial \pi_{\text{total}}}{\partial P} = 0$

$\frac{\partial \pi}{\partial P} = 25(10 - P)(-1) + 35\big((P - 2)(-1) + (10 - P)(1)\big)$

$2 - P + 10 - P$

$\frac{\partial \pi}{\partial P} = 0 \Rightarrow 25(10 - P) = 35(12 - 2P)$

$250 - 25P = 450 - 70P$

$45P = 170$

$P = 3.777$ # – per drink price

entry fee per person $= \frac{1}{2}(10 - P)\left(\frac{10 - P}{2}\right)$

$= \frac{1}{2}(10 - 3.78)\left(\frac{10 - 3.78}{2}\right)$

$= 9.67$ #