Assignment 1
Assigned on Feb 8th, 2022. Due date Saturday, Feb 26th, 2022 before midnight.

Kornchanok Vuttipakdee
6304640516

Instructions

(1) Please read the instruction carefully. Also take this habit with you into the exam room.
(2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at lease a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
(3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

(1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
(2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
(3) Default decimal point is 4.
(4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
(5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
(6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

(1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
(2) Login to BE Moodle, head into the course, then the assignment topic.
(3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

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For all questions, answer up to 4 decimal places

Question 1. (15 points) Given this information

$$n = 18 \qquad \sum_{i=1}^{n} X_i = 388.00 \qquad \sum_{i=1}^{n} Y_i = 50.90$$

$$\sum_{i=1}^{n} (X_i)^2 = 9{,}620.00 \qquad \sum_{i=1}^{n} X_i Y_i = 1{,}254.90$$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = 211.00 \qquad \sum_{i=1}^{n} (Y_i - \bar{Y})^2 = 2.5844$$

$$\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) = 20.58 \qquad \sum_{i=1}^{n} \hat{u}_i^{\,2} = 0.5781$$

Use the above sample information to answer all the following questions. Show explicitly all formulas and calculations.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$, **find the estimators** of β_1 and β_2 with OLS method. Interpret the intercept and slope coefficients.

b) Compute the value of R^2 and explain its meaning.

c) If $X_i = 30$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Calculate the estimators of $\text{var}\left(u_i\right)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$.

e) What are the 90-percent confident intervals for β_2 ? Interpret the meaning.

f) Test the hypothesis whether the slope coefficients are different from zero at 0.05 level of significance.

Question 2. Using the 2015 Health and Welfare Survey from the National Statistical Office, a simple linear regression is modeled as follows,

$$outp_i = \beta_1 + \beta_2 age_i + u_i$$

where $outp_i$ is how many times person i has visited hospital in 2015, from 0 to 7 times

age_i is how old is person i, from 0 to 97 years.

We assume that both $outp_i$ and age_i are continuous, the estimation results in the following table. Answer the following questions and show your work.

```
      Source |       SS           df       MS      Number of obs   =    27,886
-------------+----------------------------------   F(1, 27884)     =    186.96
       Model |  77.5444409         1  77.5444409   Prob > F        =    0.0000
    Residual |  11565.0627    27,884  .414756231   R-squared       =    0.0067
-------------+----------------------------------   Adj R-squared   =    0.0066
       Total |  11642.6072    27,885  .417522223   Root MSE        =    .64402

------------------------------------------------------------------------------
        outp | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .0031338   .0002292                      .0026846    .003583
       _cons |   .4279898   .0140339      Omitted         .4004828    .4554969
------------------------------------------------------------------------------
```

a) Test if both parameters are significantly different from zero or not. Use $\alpha = 0.05$.

b) Interpret the meaning of $\hat{\beta}_2$. Does the sign of $\hat{\beta}_2$ make economic sense? Explain.

c) If $outp_i$ is turned into natural logarithmic scale (ln), how would you reinterpret the relationship between $\hat{\beta}_2$ and $\widehat{outp}_i$, assumed that the given coefficient given in the table above can be used to interpret this new functional form.

d) If age_i variable is divided by 10, how does it affect both the coefficients, standard errors, and confidence intervals? Answer the changes of both the constant and slope (if there is).

e) Find the confidence interval of mean prediction at the age of 50 years old, given that $var(\hat{Y}_0) = 0.00002$ and $\alpha = 0.01$.

Question 3. Discuss in a short paragraph why the confidence interval for both the mean prediction and individual prediction get larger as the X_0 is further away from $\bar{X}$.

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1. a) $\hat{\beta}_2 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = \frac{20.58}{211} = 0.0975$ //

$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} = \frac{50.90}{18} = 2.8278$

$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{388}{18} = 21.5556$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}_i = 2.8278 - (0.0975)(21.5556) = 0.7261$ //

$\therefore Y_i = 0.7261 + 0.0975 X_i$

This means that when $X_i = 0$, $Y_i = 0.7261$ (intercept)
and with 1 unit increase of X_i, there will be 0.0975 unit increase in Y_i

b) $R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \hat{u}_i^2}{\sum(Y_i - \bar{Y})^2} = 1 - \frac{0.5781}{2.5844} = 0.7763$ //

This means that 77.63 % of $\bar{Y}$ variation can be described by X

c) $\hat{Y}_i = 0.7261 + 0.0975(30) = 3.6511$ //
This means that if $X_i = 30$, Y_i will be 3.6511 on average

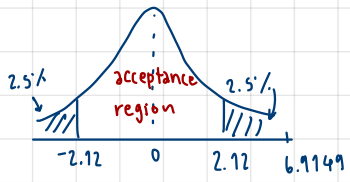
d) $\text{var}(u_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-k} = \frac{0.5781}{18-2} = 0.0361$ //

$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2 \sigma^2}{n \sum x_i^2} = \frac{9620}{(18)(211)} \times 0.0361 = 0.9144$ //

$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{0.0361}{211} = 0.0002$ //

e) $\alpha = 0.1$
$se(\hat{\beta}_2) = \text{var}(\hat{\beta}_2) = \sqrt{0.0002} = 0.0141$
$t_{\alpha/2} = t_{0.05} = 1.746$
$Pr(0.0975 - [(1.746)(0.0141)] \leq \beta_2 \leq 0.0975 + [(1.746)(0.0141)]) = 1 - 0.1$
$Pr(0.0729 \leq \beta_2 \leq 0.1221) = 0.90$ //
This means that 90% of the time, β_2 will be between 0.0729 and 0.1221

f) #1 $H_0 : \beta_2 = 0$
$H_a : \beta_2 \neq 0$
#2 $\alpha = 0.05$
#3 $se(\hat{\beta}_2) = 0.0141$
$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{se\hat{\beta}_2} = \frac{0.0975 - 0}{0.0141} = 6.9149$



#4 Upper bound : $t_{\alpha/2} = t_{0.025(16)} = 2.12$
Lower bound : $-t_{\alpha/2} = -t_{0.025(16)} = -2.12$
#5 t_{cal} lies beyond CI ($6.9149 > 2.12$). Therefore, we can reject the null hypothesis.
This means that 95% of the time, β_2 is not zero.

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2. a) β_1 :

#1 $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

#2 $\alpha = 0.05$

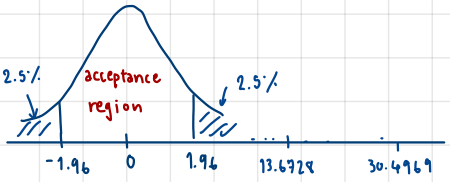
#3 $t_{cal} = \dfrac{\hat{\beta}_1 - \beta_1}{se\hat{\beta}_1} = \dfrac{0.4279898 - 0}{0.0140339} = 30.4969$

#4 Upper bound : $t_{\alpha/2} = t_{0.025\,(27,884)} = 1.96$

Lower bound : $-t_{\alpha/2} = -1.96$

#5 t_{cal} > upper bound $(30.4969 > 1.96)$. Therefore, we can reject null hypothesis.

This means that 95% of the time, β_1 is not zero.



β_2 :

#1 $H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

#2 $\alpha = 0.05$

#3 $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{se\hat{\beta}_2} = \dfrac{0.0031338 - 0}{0.0002292} = 13.6728$

#4 Upper bound : $t_{\alpha/2} = t_{0.025\,(27,884)} = 1.96$

Lower bound : $-t_{\alpha/2} = -1.96$

#5 t_{cal} > upper bound $(13.6728 > 1.96)$. Therefore, we can reject null hypothesis.

This means that 95% of the time, β_2 is not zero.

b) $\hat{\beta}_2 = 0.0031338$ means that when people become 1 year older, they go to hospital more by around 0.0031 times.

The sign of $\hat{\beta}_2$ makes economic sense as the older people become, the more chance of visiting hospital.

c) Natural logarithmic scale (ln) : $\ln \widehat{outp}_i = \hat{\beta}_1 + \hat{\beta}_2 age_i$

Relationship between $\hat{\beta}_2$ and $\widehat{outp}_i$ will be that when people age 1 yr. more, their hospital visits will increase by 0.3134% $(\hat{\beta}_2 \times 100)$

d) The coefficient, standard errors and confidence interval will scaled up by 10.

coefficient becomes 0.031338

standard errors becomes 0.002292

confidence interval becomes 0.02846 and 0.03583

e) $\widehat{outp}_i = \beta_1 + \beta_2 age_i$

$\hat{Y}_0 = 0.429 + 0.0031\,(50) \quad 0.583$

$se(\hat{Y}_0) = \sqrt{var(\hat{Y}_0)} = \sqrt{0.00002} = 0.0045$

$\alpha = 0.01$

$t_{\alpha/2} = t_{0.005} = 2.576$

$Pr\left(0.583 - [(2.576)(0.0045)] \leq Y_0 \leq 0.583 + [(2.576)(0.0045)]\right) = 1 - 0.01$

$Pr\,(0.5714 \leq Y_0 \leq 0.5946) = 0.99$

3. As $var(\hat{Y}_0) = \sigma^2 \left[\dfrac{1}{n} + \dfrac{(X_0 - \bar{X})^2}{\sum (x_i - \bar{X})^2}\right]$, if X_0 is further away from $\bar{X}$ (increase in $X_0 - \bar{X}$), the variance will be larger. As a result, standard error will be larger as well.

Moreover, the further away from $\bar{X}$, there are less data points available. Therefore, confidence interval must be larger in order to cover all datas.