Chapter 9 (2)

Utility Maximization Subject to Budget Constraint

EX: Cobb-Douglas Utility Function

The objective function: $U(x,y) = x^{\alpha}y^{\beta}$

Budget constraint: $B = p_x x + p_y y$

take ln: $\ln U = \alpha\ln(x)+\beta\ln(y)$

MRS = dy/dx (-ratio of marginal utilities) is strictly decreasing in x. → Indifference curve is convex.

The Lagrangian function: $\mathbf{L} = \alpha\ln(x) + \beta\ln(y) + \lambda(B - p_x x - p_y y)$

FOC:

$$\partial\mathbf{L}/\partial x = \alpha/x - \lambda p_x = 0$$

$$\partial\mathbf{L}/\partial y = \beta/y - \lambda p_y = 0$$

$$\partial\mathbf{L}/\partial\lambda = B - p_x x - p_y y = 0$$

$$\alpha y/\beta x = p_x/p_y$$

$$p_y = \beta p_x x / \alpha y$$

Substitute $p_y = \beta p_x x / \alpha y$ into the budget constraint: $B = p_x x + [\beta/\alpha]p_x x = ((\alpha + \beta)/\alpha)p_x x$

Solving for x*:

$x^* = (\alpha/(\alpha + \beta))B/p_x$ → demand function for good x

Find p_x and substitute p_x into the budget constraint and then solve for y*

$y^* = (\beta/(\alpha + \beta))B/p_y$ → demand function for good y