TU 152 Practice Problems

Methods of Proofs

1. Prove that if $\left[\sim(s\wedge r)\vee s\vee p\right]\rightarrow(s\wedge t)$, then t.
2. Prove that if $a\vee b\vee c$, $\sim(d\wedge e)\rightarrow\sim(f\vee a)$, and $(c\rightarrow a)\wedge(\sim b\vee f)$, then d.
3. Prove that if $\sim(p\rightarrow q)$, $(p\wedge r)\rightarrow(s\rightarrow q)$, and $(\sim r\rightarrow q)\vee(p\rightarrow s)$, then $r\leftrightarrow\sim s$.
4. Prove that if x is positive odd integer then 4 divides x^2-1.
5. Prove that $\left[(s\rightarrow t)\vee\sim r\right]\leftrightarrow\left[(r\wedge s)\rightarrow(t\vee\sim r)\right]$. Do not use Truth Table.

Mathematical induction

1. Prove that for every positive integer *n*

$$\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

2. Prove that for every positive integer *n* that is greater than 4,

$$4^n>n^4.$$

3. For every nonnegative integer n,

$$3\,|\left(2^{2n}-1\right)$$

Inequalities

Solve the following inequalities.

1. $x+1>x-2$
2. $\frac{2x}{3}-3\geq4+\frac{2x}{3}$
3. $1-\frac{x}{2}\geq2-\frac{3x}{2}$
4. $3x<\frac{x}{3}-2$
4. $x^2-5x+4\geq0$
6. $x^3+3x^2+2x>0$
7. $\left(4x^2-2x+3\right)\left(2x^2-3x-20\right)\leq0$
8. $\frac{x^2}{x^2-3x+2}\geq1$
9. $\frac{2x^2-3x+5}{x+1}\leq x$
10. $1<\frac{5}{x^2-x+3}$
11. $\frac{-x^2+2x}{(x-2)\left(4x^2-2x+1\right)}<0$
12. $\frac{2x^2-4x+16}{x+2}\leq4$
13. $\frac{5x^2+7x-1}{x+1}<x+2$
14. $\frac{x^3+2x^2+5x}{x^2+2x-3}\geq0$