Example Questions for Multiple Integrals

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1. Determine $\int_0^1 \int_{x^3}^{x^2} (x^3 - xy)\, dy\, dx$ Ans: $\frac{1}{336}$

2. Determine $\int_{-1}^{1} \int_{0}^{1-x} (2x+1)\, dy\, dx$ Ans: $\frac{2}{3}$

3. Find $\int_{1}^{\ln 2} \int_{e^y}^{2} dx\, dy$ Ans: $\ln 4 - 4 + e$

4. Determine $\int_{-1}^{0} \int_{-1}^{2} \int_{1}^{2} 6xy^2 z^3\, dx\, dy\, dz$ Ans: $-\frac{27}{4}$

5. Determine $\int_0^1 \int_0^x \int_0^{x-y} x\, dz\, dy\, dx$ Ans: $\frac{1}{8}$

6. Determine $\int_0^1 \int_{x^2}^{x} \int_0^{xy} dz\, dy\, dx$ Ans: $\frac{1}{24}$

7. Determine $\int_0^2 \int_0^2 e^{-(x+y)}\, dx\, dy$ Ans: $e^{-4} - e^{-3} - e^{-2} + e^{-1}$

8. Determine $\int_0^4 \int_{y/2}^{2} xy\, dx\, dy$ Ans: 8

9. Determine $\int_0^1 \int_{\sqrt{x}}^{x^2} 7(x^2 + 2xy - 3y^2)\, dy\, dx$ Ans: $\frac{1}{30}$

10. Determine $\int_0^1 \int_1^2 \frac{x}{y^2} e^{x/y}\, dy\, dx$ Ans: $e - 2e^{\frac{1}{2}} + 1$

11. Determine $\iint \frac{x}{y} e^{x/y}\, dA$ where $0 \le x \le 1$ and $1 \le y \le 2$
Ans: $-\frac{1}{y^2} e^{\frac{1}{2}}$

Example Questions for Differential Eqs.

NO.
DATE

Solve the differential equations, determine stability and prove that the answer is correct

1. Solve $\frac{dy}{dt} + 2y = 6$ and $y(0) = 10$

   Ans: $y = 3 + 7e^{-2t}$ stable/ Converge to 3

2. $\frac{dy}{dt} + 4y = 0$ and $y(0) = 1$

   Ans: $y = e^{-4t}$ stable/converge to zero.

3. $\frac{dy}{dt} + 3t^2 y = 0$

   Ans: $y = Ke^{-t^3}$ stable/converge to zero.

4. $\frac{dy}{dt} + 2ty = t$

   Ans: $IF = e^{t^2}$, $y = \frac{1}{2} + C_2 e^{-t^2}$ stable / converge to $\frac{1}{2}$.

5. $\frac{dy}{dt} + 2ty = 4t$

   Ans: $y = Ae^{-2t^2} + 1$ stable/converge to 1

6. $\frac{dy}{dt} + 4y = 12$ and $y(0) = 2$

   Ans: $y = -e^{-4t} + 3$ stable / converge to 3

7. $\frac{dy}{dt} - 2y = 0$ and $y(0) = 9$

   Ans: $y = 9e^{2t}$ unstable/diverge

8. $\frac{dy}{dt} + y = 4$ and $y(0) = 0$

   Ans: $y = 4(1 - e^{-t})$ stable / converge to 4

9. $\frac{dy}{dt} - 5y = 0$ and $y(0) = 6$

   Ans: $y = 6e^{5t}$ unstable / diverge

10. $\frac{dy}{dt} - 7y = 7$ and $y(0) = 7$

    Ans: $y = 8e^{7t} - 1$ unstable / diverge

11. $\frac{dy}{dt} + 2ty = t$ and $y(0) = \frac{3}{2}$

    Ans: $y = Ae^{-5t} + 3$ stable/converge to 3

12. $2\frac{dy}{dt} + 12y + 2e^t = 0$ and $y(0) = \frac{6}{7}$

    Ans: $y = e^{-6t} - \frac{1}{7}e^t$ unstable / diverge

13. $y''(t) + y'(t) - 2y = -10$

    Ans: $y(t) = A_1 e^t + A_2 e^{-2t} + 5$

    If $y(0) = 12$, $y'(0) = -2$

    ans: $y(t) = 4e^t + 3e^{-2t} + 5$

    unstable / diverge

NO.

DATE

14. $\ddot{y} + 6\dot{y} + 9y = 27$ Ans: $y = (A+Bt)e^{-3t} + 3$

If $y(0) = 5$, $\dot{y}(0) = -5$ Ans: $y = (2+t)e^{-3t} + 3$

15. $\ddot{y} + 3\dot{y} - 4y = 12$ Ans: $y = Ae^{t} + Be^{-4t} - 3$

If $y(0) = 4$, $\dot{y}(0) = 2$ Ans: $y = 6e^{t} + e^{-4t} - 3$

16. $\ddot{y} - 2\dot{y} + y = 3$ Ans: $y = e^{t} + te^{t} + 3$