Game Theory

EE311

Topics to be Discussed

- Introduction
  - Games and the representation of a game
  - Types of games
- Equilibrium for simultaneous games
  - Dominant Strategies
  - The Nash Equilibrium
  - Maximin Equilibrium
- Repeated Games
- Sequential Games
  - Threats, Commitments, and Credibility
  - Entry Deterrence

1. Introduction

- Game is any situation in which players (the participants) make strategic decisions that can affect each other.
  - Ex: firms competing with each other by setting prices
- We are not interested in games that outcome is solely determined by luck.

Components of a game

- Players
- Strategy: a rule or plan of action for playing the game
- Payoff: outcomes of the game that generate rewards or benefits

Assumptions

- Perfect information: each player know the rules and payoffs.
  - "I know that you know that I know."
- Rational players– they think through their actions.
  - "If I believe that my competitors are rational and act to maximise their own profits, how should I take their behaviour into account when making my own profit-maximising decisions?"

Representation of a game

- Normal or Payoff matrix form

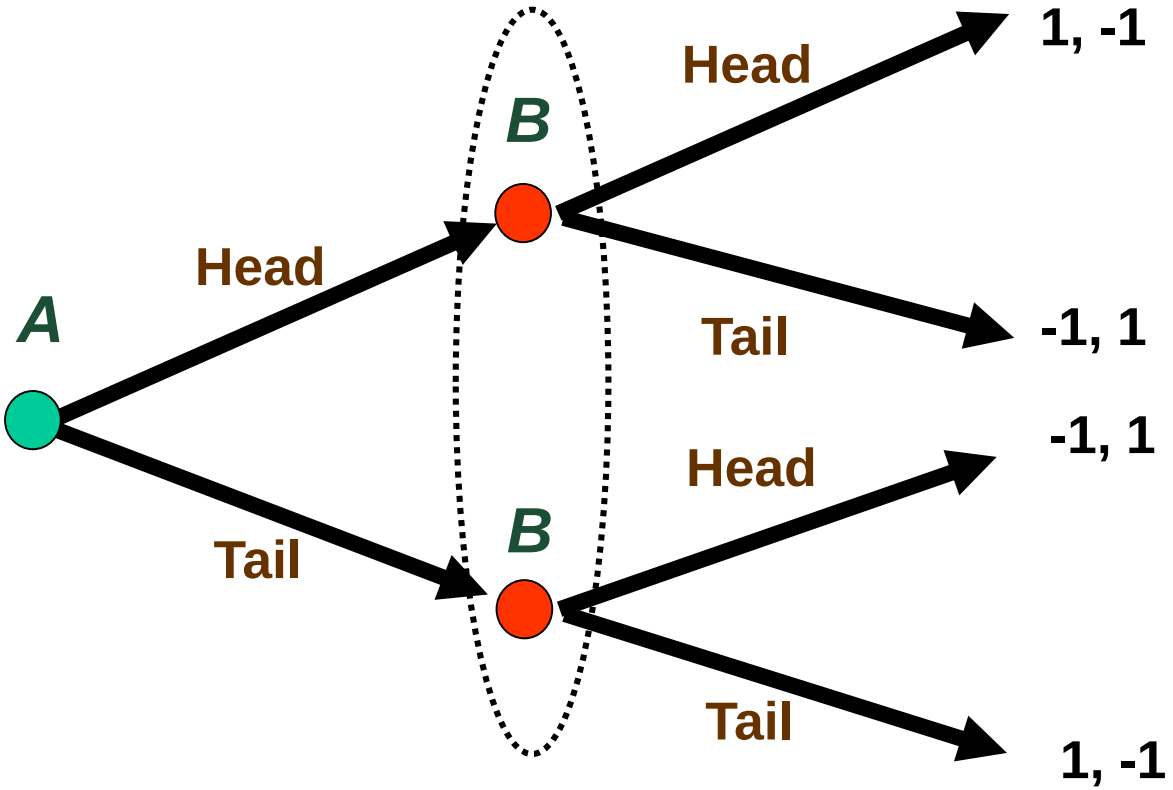
Matching Pennies Game

		Player B: Head	Player B: Tail
Player A:  Head		1, -1	-1, 1
Player A:  Tail		-1, 1	1, -1

Representation of a game

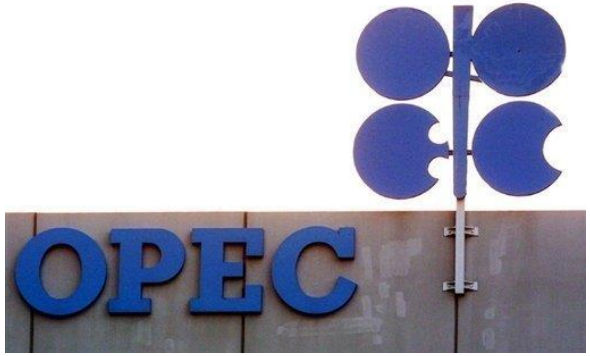
- Extensive form or game tree

Matching Pennies Game

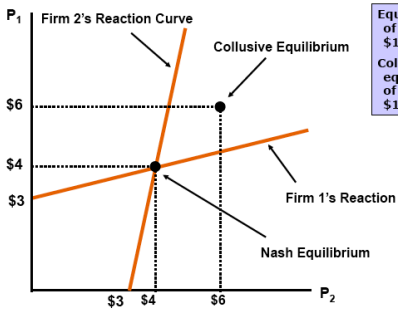


Types of Games: Noncooperative v. Cooperative Games

- Cooperative Game
  - Players negotiate binding contracts that allow them to plan joint strategies
    - Example: Cartel
    - Binding contracts are possible



- Noncooperative Game
  - Negotiation and enforcement of binding contracts between players is not possible
    - Example: Bertrand competition
    - Binding contracts are not possible



Simultaneous vs. Sequential games

- Simultaneous games
  - All players compete at the same time
  - Example: Cournot or Bertrand competition, Paper-Scissors-Hammer
- Sequential games
  - One player moves first then follows by another player
  - Example: Poker, Stackelberg, Price leadership

One-shot vs. Repeated games
Constant sum vs. Non-constant sum games

- One-shot games
  - The game is played only once; e.g., FA Cup
- Repeated games
  - The game is played many times; e.g., business
- Constant sum
  - The sum of all payoffs are constant; e.g., gambling
- Non-constant sum games
  - The sum of all payoffs can vary; e.g., oligopoly models, free trade agreements

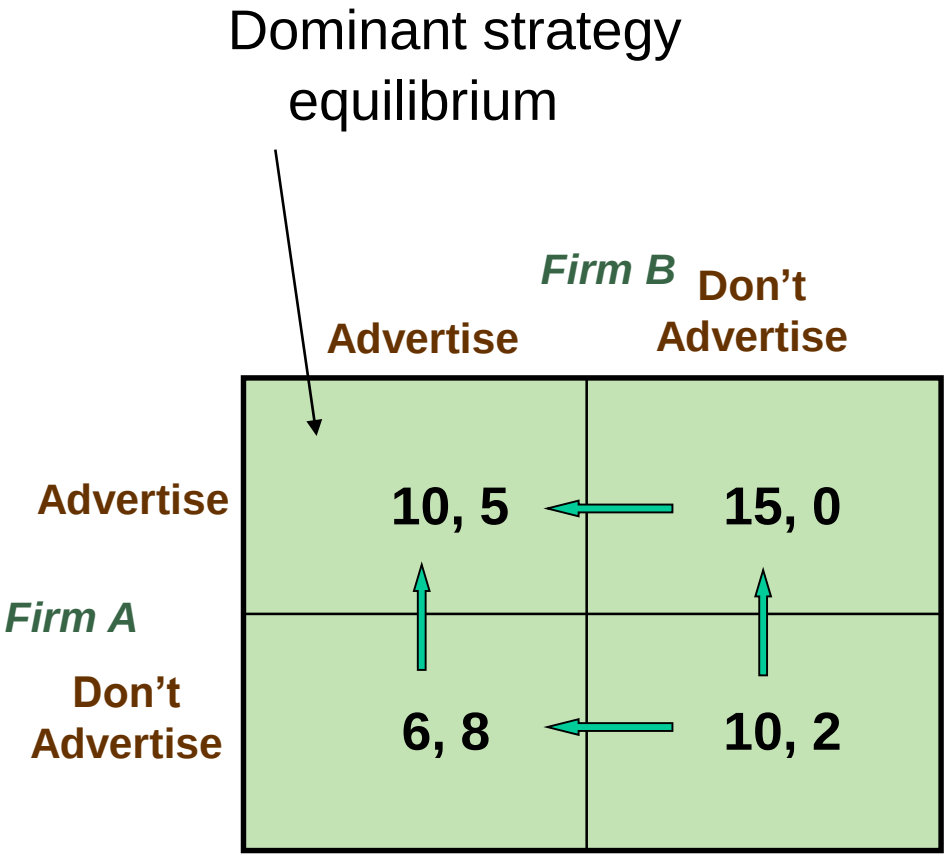
2. Equilibrium for one-shot and simultaneous games

1. Dominant Strategies

- Dominant Strategy is the one that is optimal no matter what an opponent does.
- Dominated Strategy is the one that always gives the lowest pay off no matter what an opponent does.
  - The player will not use the dominated strategy and the opponent also know this fact.
  - It can be eliminated from the payoff matrix

Payoff Matrix for Advertising Game

- Example
  - A: regardless of B, advertising is the best
  - B: regardless of A, advertising is best
  - Eliminate all the dominated strategies
  - What is left is the dominant strategy equilibrium

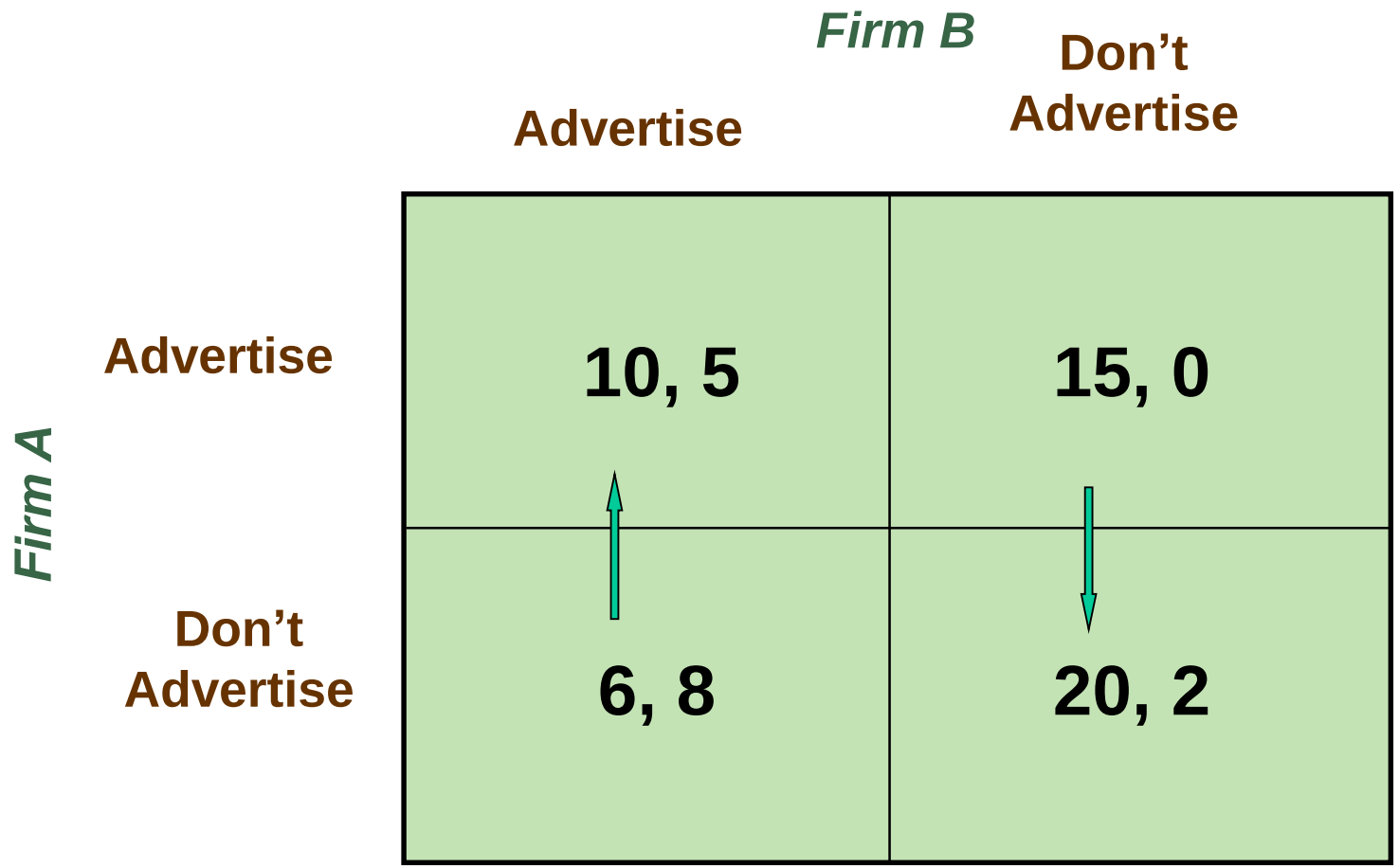


Dominant Strategies

- Equilibrium in dominant strategies
  - Outcome of a game in which each firm is doing the best it can, regardless of what its competitors are doing
  - Optimal strategy is determined without worrying about actions of other players
- However, not every game has a dominant strategy for each player

Modified Advertising Game

No dominant strategy



Finding an equilibrium for one-shot and simultaneous games: The Nash Equilibrium

- A dominant strategy is stable, but in many games, one or more party does not have a dominant strategy.
- A more general equilibrium concept is the Nash Equilibrium
  - A set of strategies (or actions) such that each player is doing the best it can given the actions of its opponents

The Nash Equilibrium vs. dominant strategy equilibrium

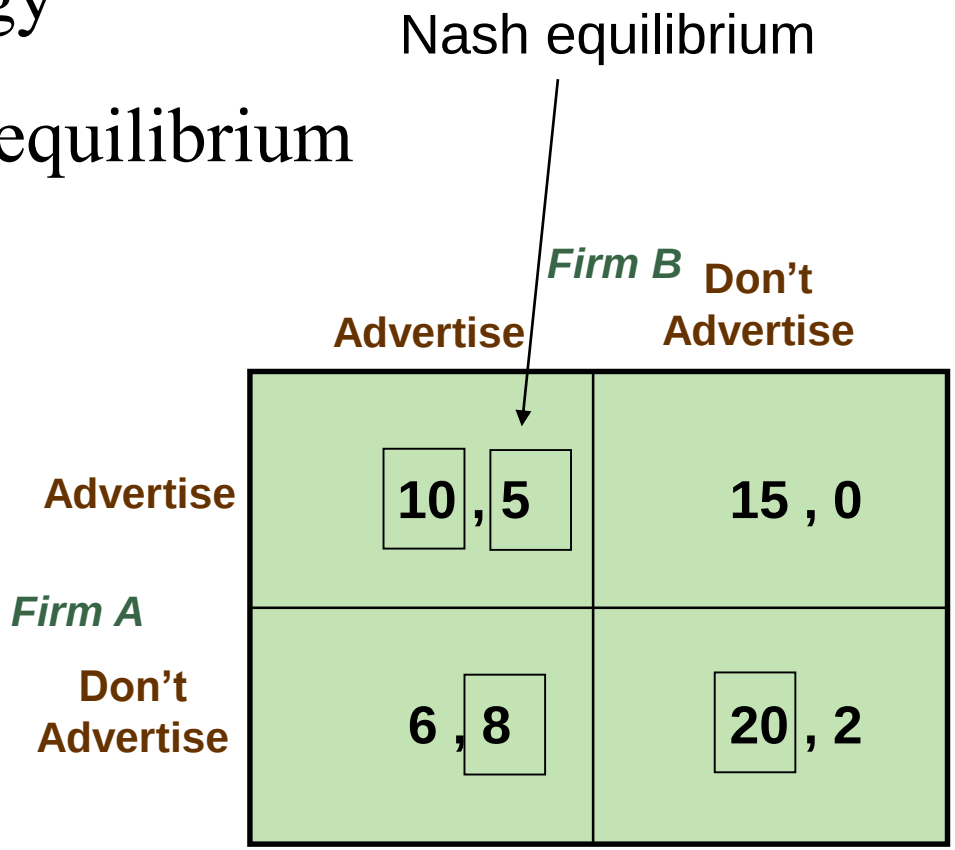
- Dominant Strategy
  - "I'm doing the best I can no matter what you do. You're doing the best you can no matter what I do."
- Nash Equilibrium
  - "I'm doing the best I can given what you are doing. You're doing the best you can given what I am doing."
- Dominant strategy is a special case of Nash equilibrium

Steps to find Nash equilibrium

1. Assume one rival strategy, then choose the strategy that yields the best payoff. Mark it.
2. Move on to the next rival strategy, repeat step one again until all rival strategies are exhausted.
3. Repeat step 1 and 2 for the other side.
4. The cell that has two marks on is a Nash equilibrium.

Modified Advertising Game revisit

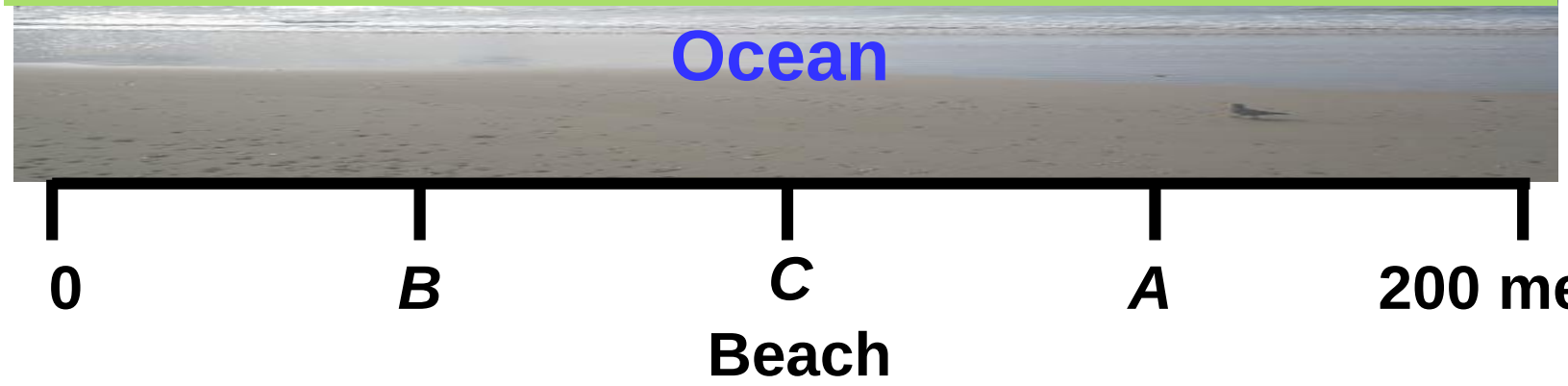
- No dominant strategy
- But there is a Nash equilibrium



Example: Beach Location Game

- Scenario
  - Two competitors, Y and X, selling soft drinks
  - Beach 200 meters long
  - Sunbathers are spread evenly along the beach
  - Price Y = Price X
  - Customer will buy from the closest vendor

Beach Location Game

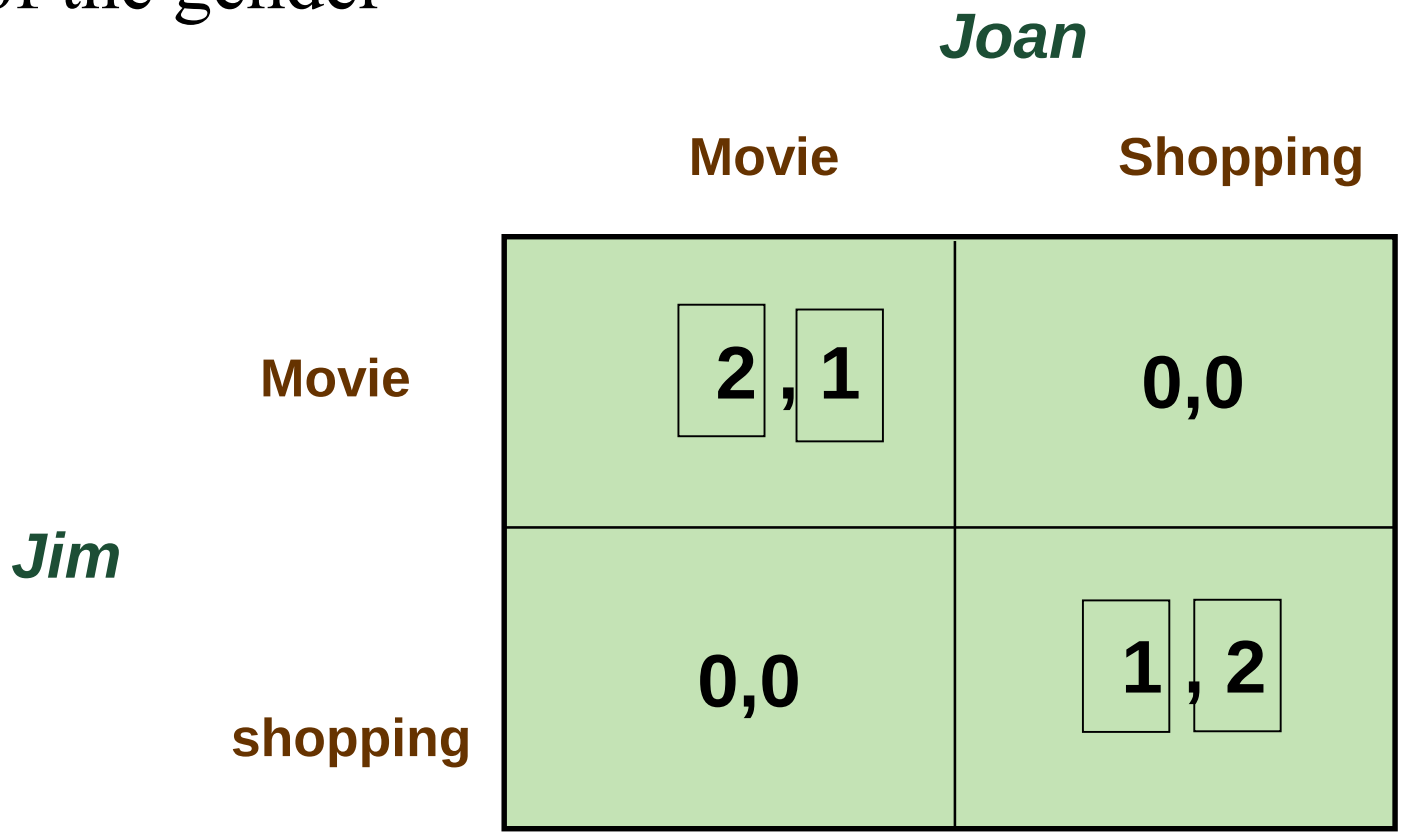


- Where will the competitors locate (i.e. where is the Nash equilibrium)?
- Will want to all locate in center of beach.
  - Gas stations or department stores at the intersections, clusters of restaurants, etc.
  - Varieties of goods cluster very close to the tastes of the median population.

Remarks

1. Multiple equilibria:

Battle of the gender



Remarks

2. Inefficient outcome:

Prisoner's dilemma

- Two prisoners have been accused of collaborating in a crime.
- Policemen have only some evidence to convict them.
- They are in separate jail cells and cannot communicate.
- Each has been asked to confess.

Remarks

2. Inefficient outcome:

Prisoner's dilemma

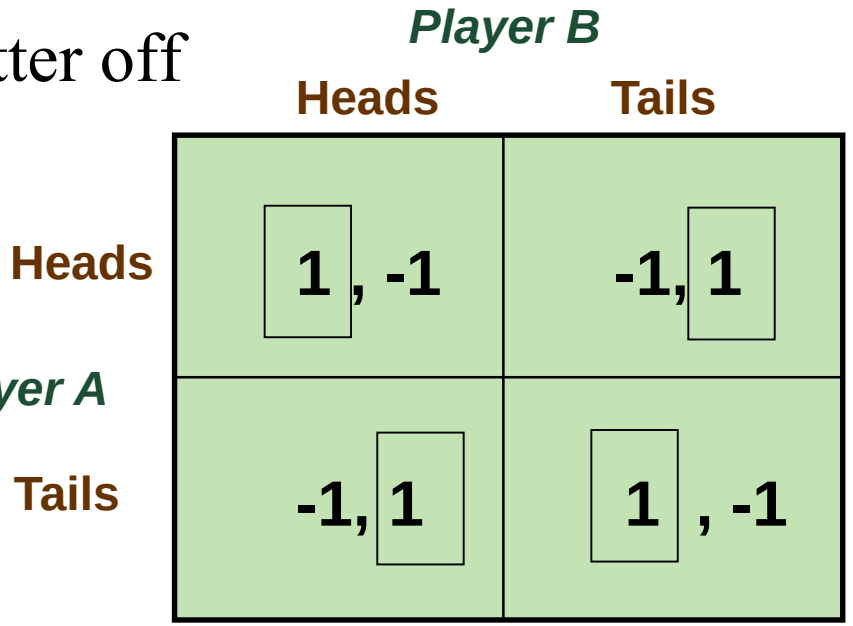
		Prisoner 2: Confess	Prisoner 2: Don't confess
Prisoner 1	Confess	-6 , -6	0 , -12
	Don't confess	-12, 0	-2 , -2

Remarks

3. No Nash equilibrium

- Pure strategy: No Nash equilibrium
  - No combination of head and tails leaves both players better off

Matching pennies



- Mixed strategy: Random choice is a Nash equilibrium

Mixed Strategy

- Pure Strategy
  - Player makes a specific choice or takes a specific action
- Mixed Strategy
  - Player makes a random choice among two or more possible actions, based on a set of chosen probabilities

Mixed strategy

- Player A might flip coin playing heads with ½ probability and tails with ½ probability.
- If both players follow this strategy, there is a Nash equilibrium – both player will be doing the best they can given what they opponent is doing.
- Although the outcome is random, the expected payoff is 0 for each player.

Mixed Strategy

- One reason to consider mixed strategies is when there is a game that do not have any Nash equilibriums in pure strategy.
- When allowing for mixed strategies, every game has a Nash equilibrium
- A firm might not find it reasonable to use mixed strategy

Remarks

4.Playing Nash strategy against an irrational rival may be risky

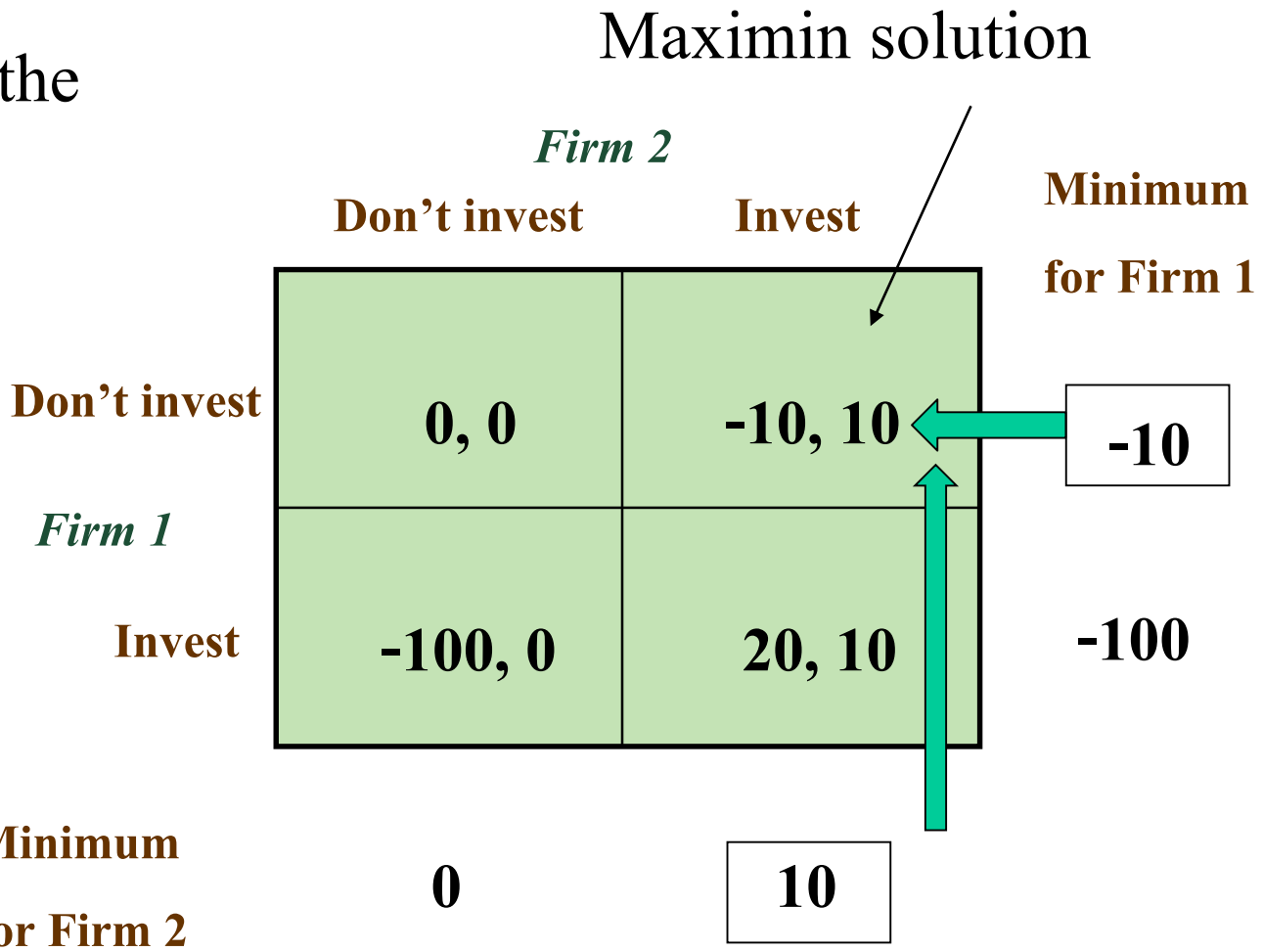
- Nash equilibrium
  - Firm 1: invest
  - Firm 2: Invest
- This assumes firm 2 understands the game and is rational
- If Firm 2 does not invest, Firm 1 incurs significant losses

Firm 1 \ Firm 2	Don't invest	Invest
Don't invest	0, 0	-10, 10
Invest	-100, 0	20, 10

Finding an equilibrium for one-shot and simultaneous games

3.Maximin Strategy

Maximize the minimum payoffs



Maximin Strategy

- If both are rational and informed
  - Both firms invest
  - Nash equilibrium
- If Player 2 is not rational or completely informed
  - Firm 1's maximin strategy is to not invest
  - Firm 2's maximin strategy is to invest.
  - If 1 knows 2 is using a maximin strategy, 1 would invest

Prisoners' Dilemma

- What is the:
  - Dominant strategy
  - Nash equilibrium
  - Maximin solution
- Dominant strategies are also maximin strategies
- Both confess is both Nash equilibrium and maximin solution

Prisoner A \ Prisoner B	Confess	Don't Confess
Confess	-6, -6	0, -12
Don't Confess	-12, 0	-2, -2

Pareto Efficient Solution

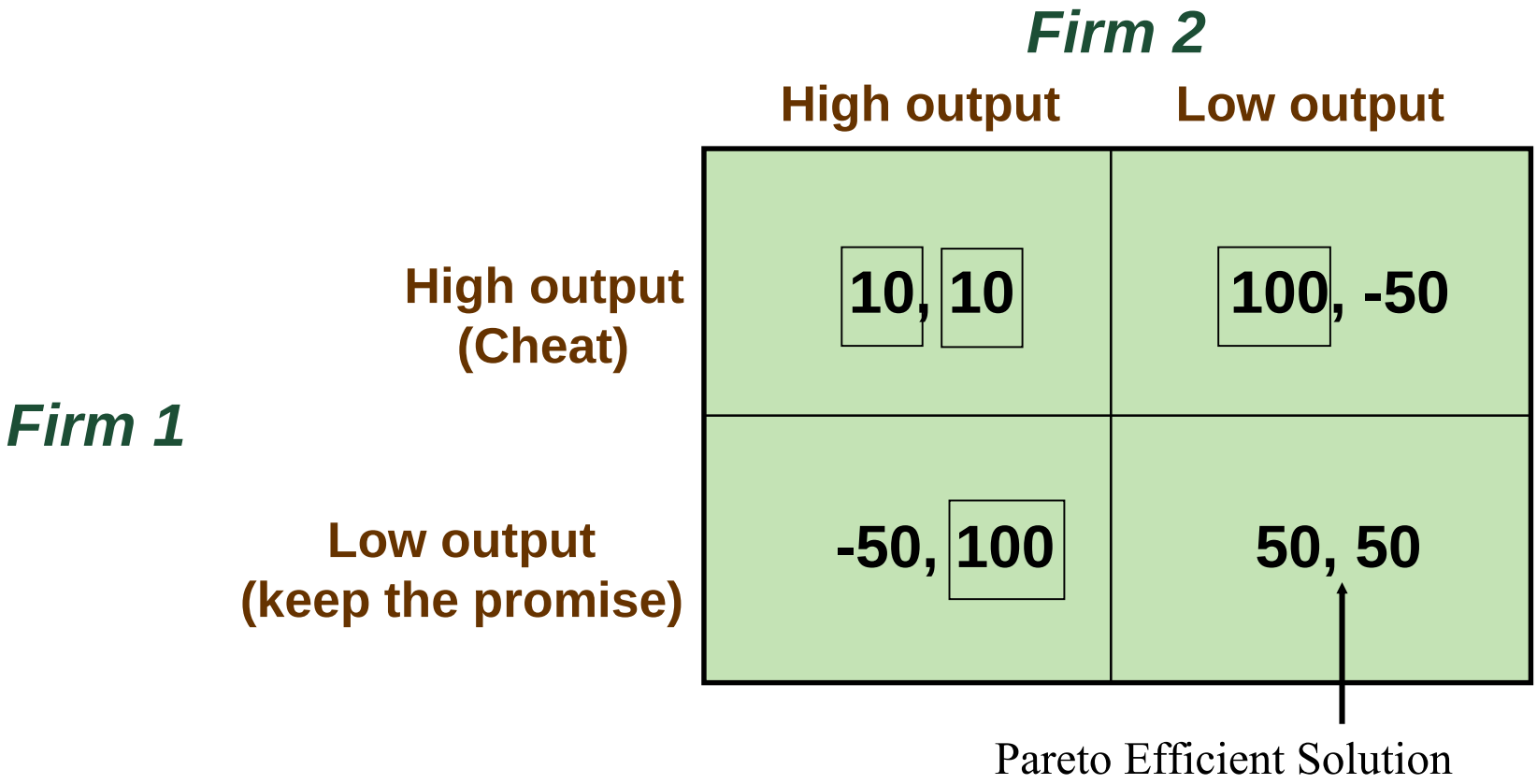
- Nash equilibrium is not Pareto efficient since at least one can be better of if they don't confess

Pareto efficiency

- Pareto improvement: an options that at least one is made better off without making someone else worse off.
- Pareto efficiency: an option that no one can be made better off without making someone else worse off
  - If it is still possible to make some one better off without hurting any one at the first place, it means that original option is not efficient yet.

Prisoner's dilemma and cartel

- The cartel members is facing a Prisoner's dilemma



3. Repeated Games

- Game in which actions are taken and payoffs received over and over again
- Oligopolistic firms play a repeated game.
- With each repetition of the Prisoners' Dilemma, firms can develop reputations about their behavior and study the behavior of their competitors.

Exercise

A cartel consists of 4 firms. They agree to set high price.

The payoff for each firm who "cheat" of "cooperate" are as follows:

# of cheating firms	$\pi_{cheating}$	$\pi_{cooperate}$
0	-	300
1	400	200
2	300	100
3	100	0
4	0	-

Example: Pricing Problem

- How does a firm find a strategy that would work best on average against all or almost all other strategies
- Tit-for-tat strategy
  - Repeated game strategy in which a player responds to an opponent's previous play, cooperating with cooperative opponents and retaliating against uncooperative ones.

Tit-for-Tat Strategy

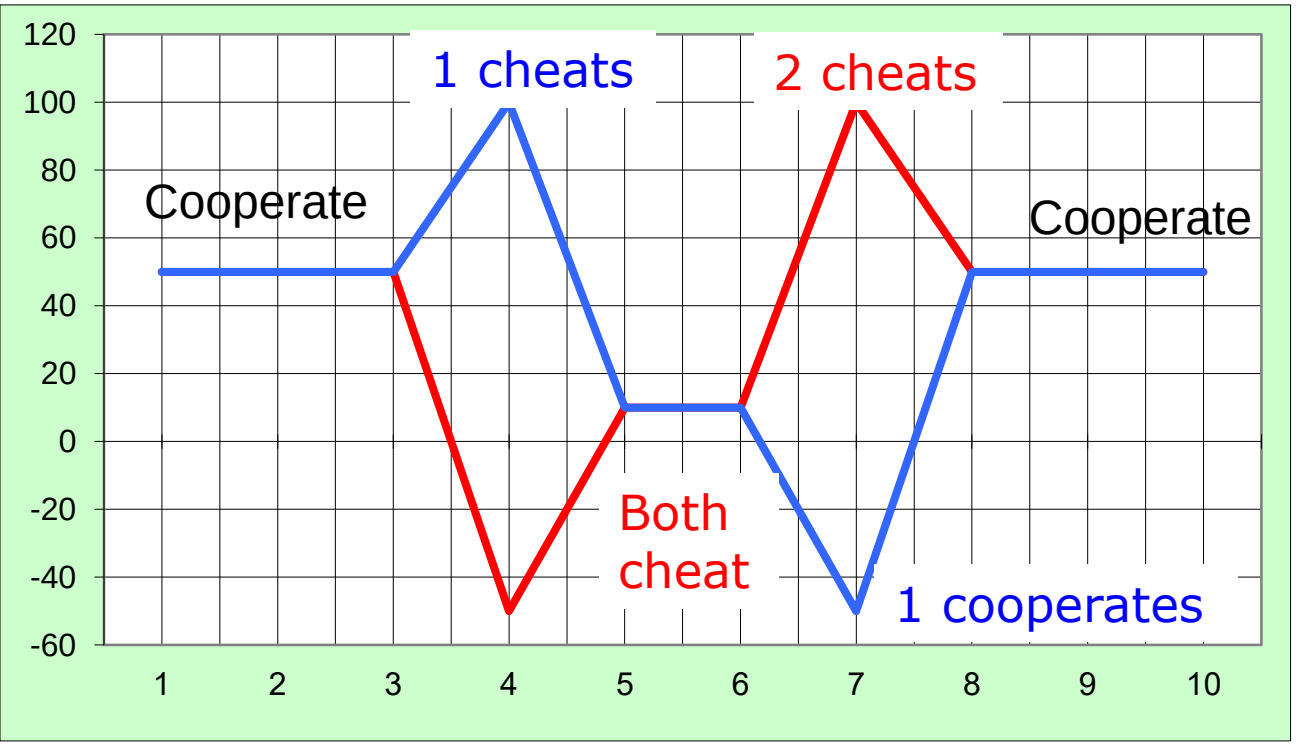
- What if the game is infinitely repeated
  - Competitors repeatedly set price every month, forever
  - Tit-for-tat strategy is rational
    - If competitor charges low price and undercuts firm
    - Competitor will get high profits that month but know I will lower price next month
    - Both of us will get lower profits if keep undercutting. Furthermore I will set high price one period after him and get high profits back.
    - So not rational to undercut.

Tit-for-Tat Strategy

- What if repeat a finite number of times
  - If both firms are rational, they will charge high prices until the last month
  - After the last month, there is no retaliation possible
  - But in the month before last month, knowing that will charge low price in last month, will charge low price in month before
  - Keep going and see that only rational outcome is for both firms to charge low price every month
- Most managers don't know how long they will be competing with their rivals
- In a repeated game, prisoners dilemma can have cooperative outcome

Tit-for-Tat Strategy: Example

- Changes in profits of firms that compete for 10 round using information from slide 32
- Firm 1 cheats at round 4 and 5



- Total profits for 1 is 370 equals to that of 2.
- If both firms cooperate for all 10 round, profits is 500.

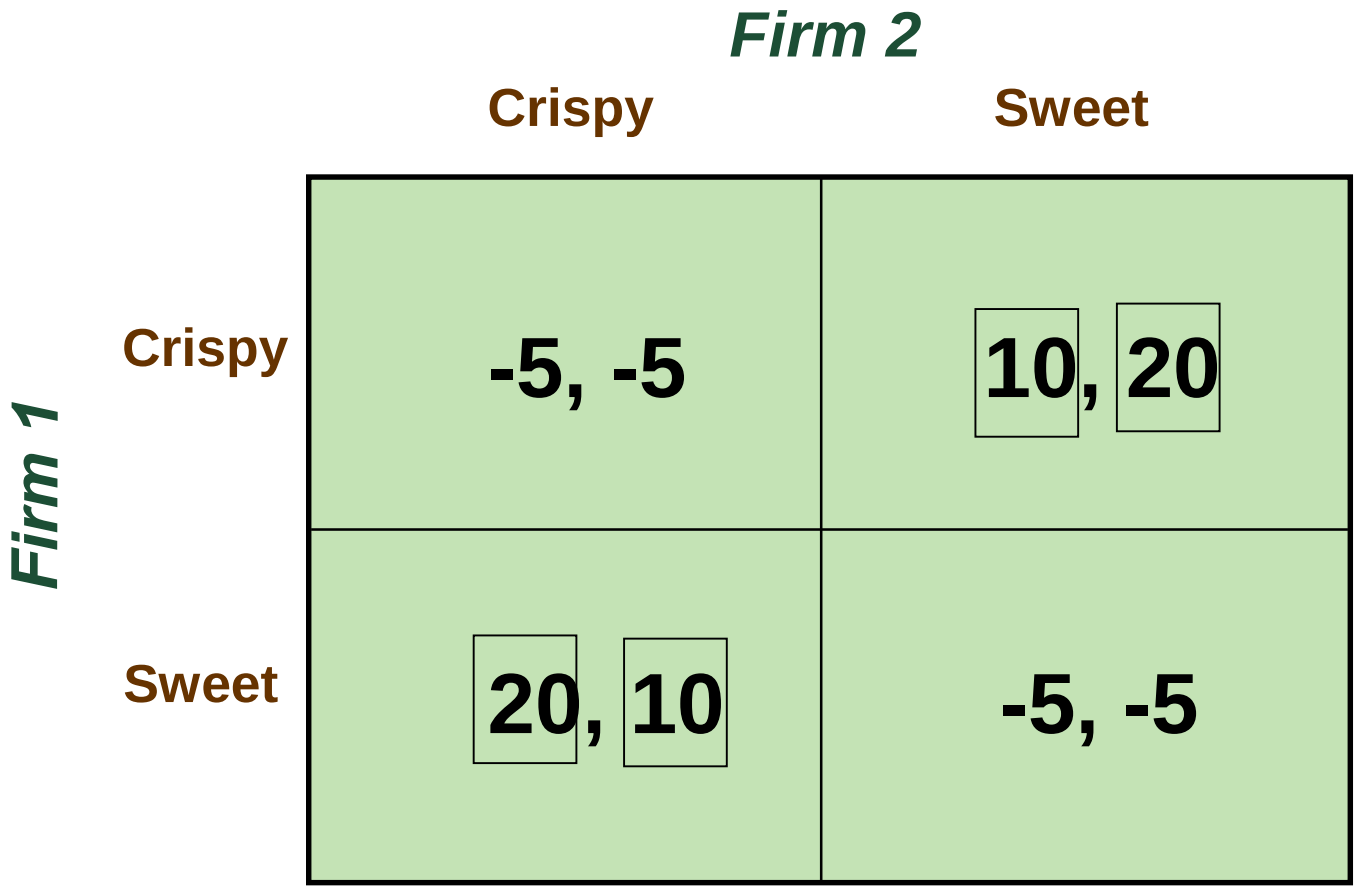
Repeated Games

- Conclusion
  - Cooperation is difficult at best since these factors may change in the long-run.
  - Need a small number of firms
  - Need stable demand and cost conditions
    - This could lead to price wars if don't have them

4. Sequential Games

- Players move in turn, responding to each other's actions and reactions
  - Ex: Chess,
  - Stackelberg model
  - Responding to a competitor's ad campaign
  - Entry decisions
  - Responding to regulatory policy

Example: Product Choice Problem



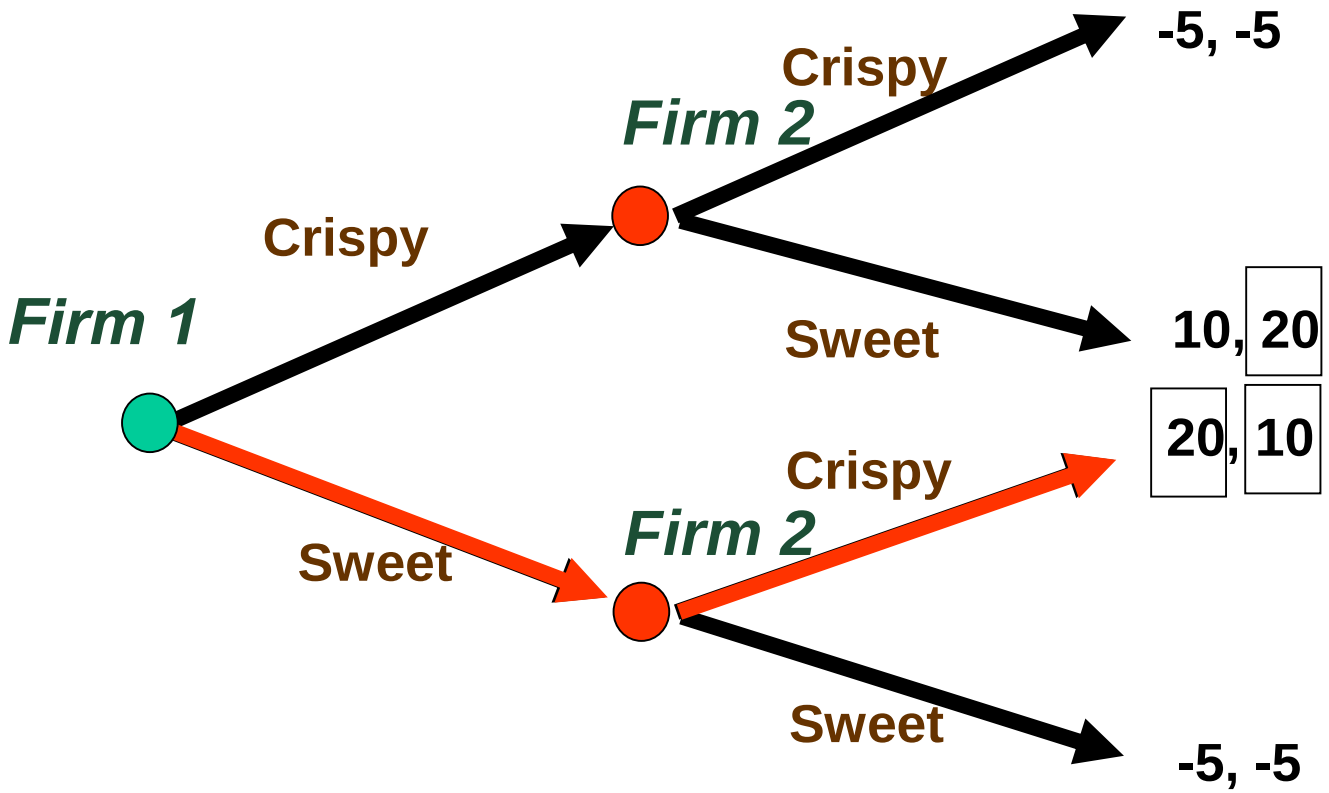
Example: Product Choice Problem

- A product choice problem
  - Two new (sweet, crispy) cereals
  - Successful only if each firm produces one cereal
  - Sweet will sell better
  - Both still profitable with only one producer

Example: Product Choice Problem

- If firms both announce their decision independently and simultaneously, they will both pick sweet cereal and both will lose money
- What if firm 1 speed up production and introduced new cereal first
  - Now there is a sequential game
  - Firm 1 think about what firm 2 will do

Product Choice Game in Extensive Form



Sequential Games

- The Advantage of Moving First
  - In this product-choice game, there is a clear advantage to moving first.
  - In the Stackelberg model, the first firm can choose a large level of output thereby forcing second firm to choose a small level.

Threats, Commitments, and Credibility

- Firm 2 in the product choice game may threat that it will produce sweet cereal no matter what firm 1 chooses?
- But firm 1 will know that it is an empty threat.
- Empty Threats
  - When firms know the payoffs of each others actions, firms cannot make threats the other firm knows they will not follow.
  - In our example, firm 2 will choose crispy cereal after firm 1 choose sweet cereal and firm 1 knows it

Threats, Commitments, and Credibility

- Strategic Move
  - Action that gives a player an advantage by constraining his behavior
  - Firm 2 must constrain his behavior to the extent Firm 1 is convinced that he is committed

Threats, Commitments, and Credibility

- How To Make the First Move
  - Demonstrate Commitment
  - Firm 2 must do more than announce they will produce sweet cereal
    - Invest in expensive advertising campaign
    - Buy large order of sugar and send invoice to firm 1
  - Commitment must be enough to induce firm 1 to make the decision firm 2 wants it to make
  - Example

Modified Production Choice Problem

- Strategic commitments can be effective but not without risk
  - Rely heavily on accurate knowledge of payoff matrix
  - May have competitors out there that they don't know about and lose sale

Role of Reputation

- If one company gets the reputation of being irrational
- Threat might be credible because irrational people don't always make profit-maximising decisions
- A party thought to be crazy can lead to a significant advantage

Entry Deterrence

- Barriers to entry important for monopoly power
  - Economies of scale, patents and licenses, access to critical inputs
  - Firms can also deter entry
- To deter entry, the incumbent firm must convince any potential competitor that entry will be unprofitable.

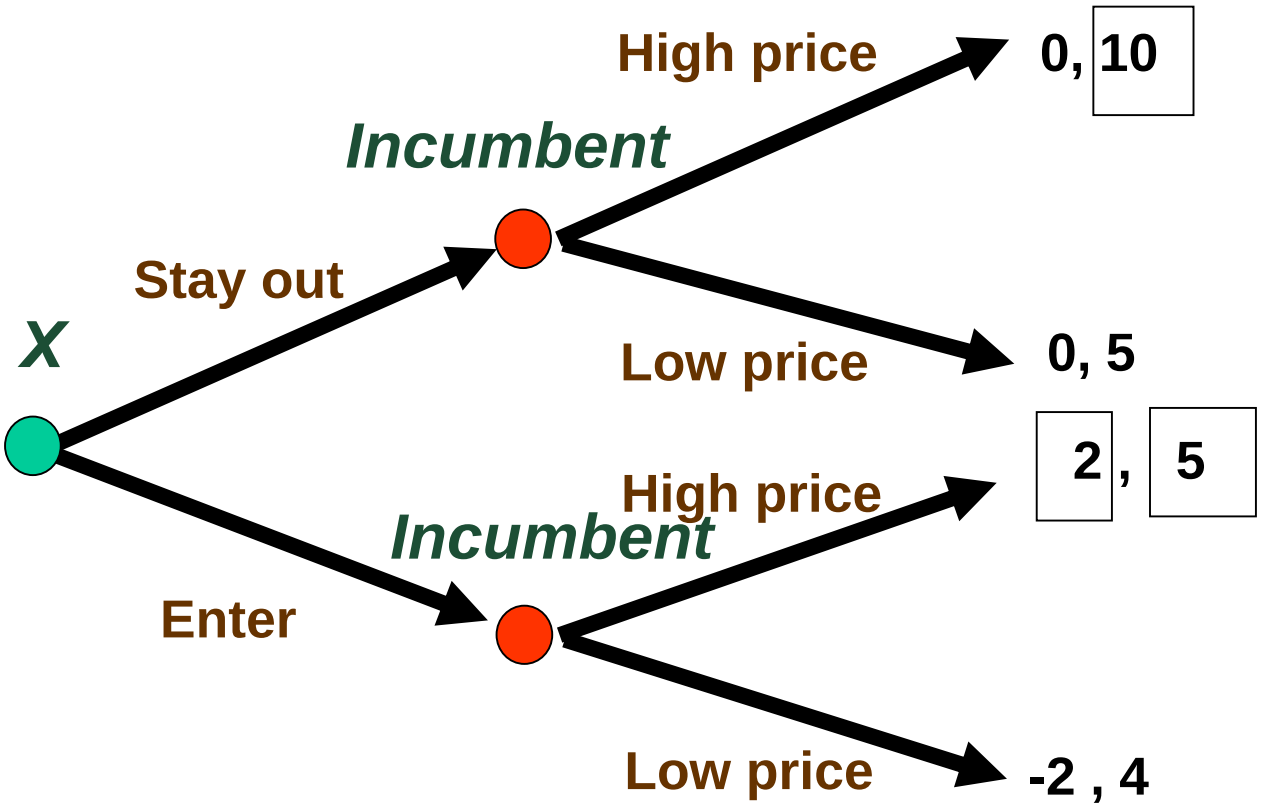
Entry Possibilities

		Incumbent
	High price (accommodation)	**Low Price (warfare)**
Enter	**2, 5**	**-2, 4**
Potential Entrant X ***($80 fixed costs)*** **Stay out**	**0, 10**	**0, 5**

Entry Deterrence

- Scenario
  - If X does not enter I makes a profit of $10 million.
  - If X enters and charges a high price I earns a profit of $5 million and X earns $2 million.
  - If X enters and I charges a low price I earns a profit of $4 million and X earns $-2 million.
- Could threaten X with warfare if enter market
  - Not credible because once X has entered, it is in your best interest to accommodate and maintain high price.

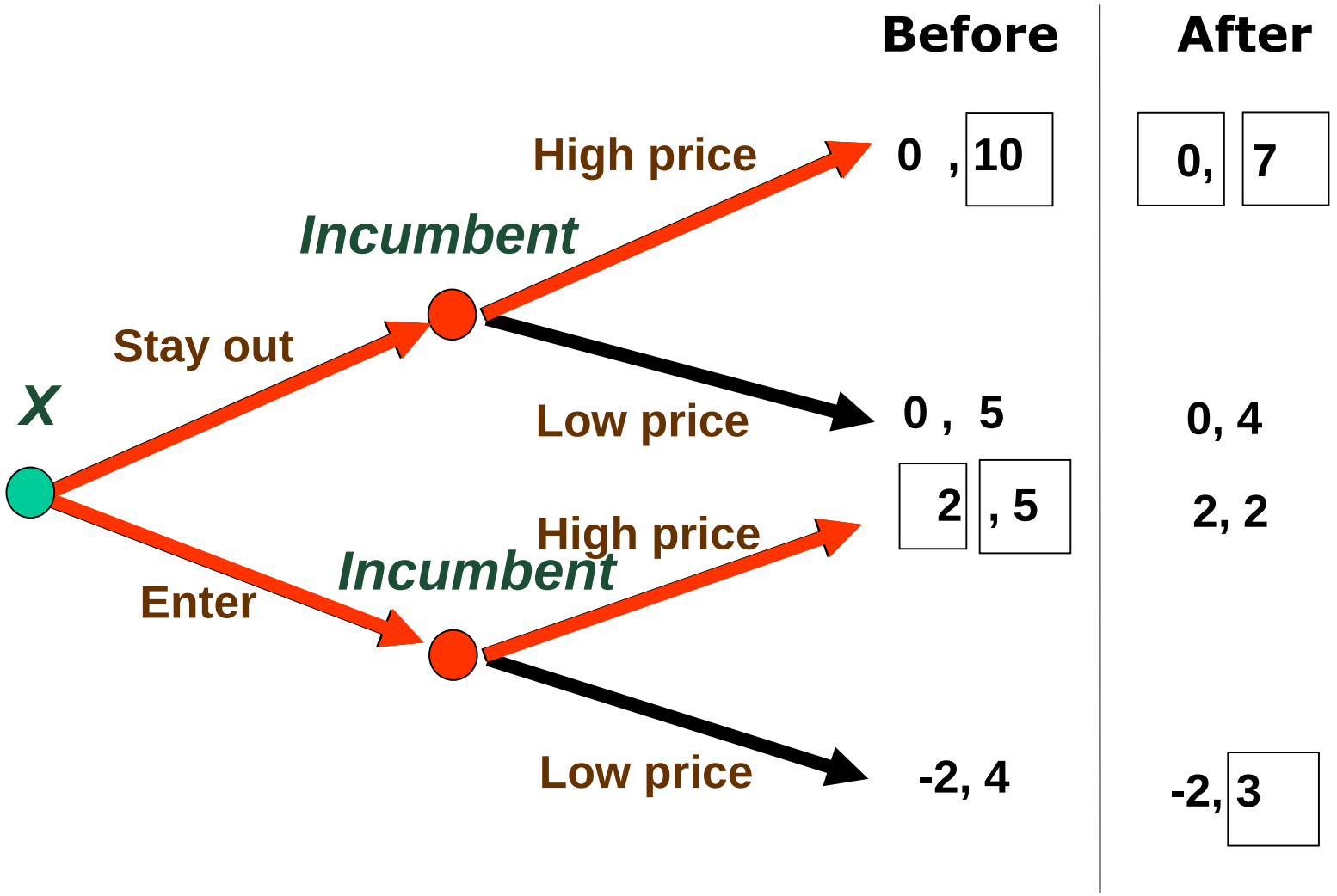
Entry Deterrence



Entry Deterrence

- What if Incumbent makes an investment before entry to increase capacity
  - Irrevocable commitment
- Gives new payoff matrix since profits will be reduced by investment
- Profits reduce less if the new capacity is used
- Threat is completely credible
- Rational for firm X to stay out of market

Entry Deterrence



Entry Deterrence

- If incumbent has reputation of price cutting competitors even at loss, then threat will be credible.
- Short run losses may be offset by long run gains as monopolist