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1. Two individuals agree at date 0 to a forward contract that matures at date 2. The contract is written on an underlying asset that pays a dividend at date 1 equal to D_1. Let f_2 be the date 2 random payoff (profit) to the individual who is the long party in the forward contract. Also let m_{0i} be the stochastic discount factor over the period from dates 0 to i where $i = 1, 2$, and let $E_0[\cdot]$ be the expectations operator at date 0. What is the value of $E_0[m_{02} f_2]$? Explain your answer.

Let P_i = Price of asset at date i

$$P_0 = E_0[m_{01} D_1] + E_0[m_{02} P_2]$$

$$= D_0 + E_0[m_{02} P_2] \quad \text{——— } (1)$$

Let F_{02} = forward price

then $f_2 = P_2 - F_{02}$

Thus, $E_0[m_{02} f_2] = E_0[m_{02}(P_2 - F_{02})]$

$$= E_0[m_{02} P_2] - E_0[m_{02} F_{02}]$$

From (1)

$$E_0[m_{02} f_2] = P_0 - D_0 - E_0[m_{02} F_{02}]$$

$$= P_0 - D_0 - E_0[m_{02}] F_{02}$$

$$= P_0 - D_0 - \frac{1}{R_f^2} F_{02}$$

with the absence of arbitrage,

$$F_{02} = R_f^2 (S_0 - D_0)$$

Then $E_0[m_{02} f_2] = 0$

2. Assume that there is an economy populated by infinitely-lived representative individuals who maximize the lifetime utility function

$$E_0\left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t}\right]$$

where c_t is consumption at date t and $a > 0$, $0 < \delta < 1$. The economy is a Lucas (1978) endowment economy having multiple risky assets paying date t dividends that total d_t per capita. Write down an expression for the equilibrium per capita price of the market portfolio in terms of the assets' future dividends.

By Lucas (1978) model,

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{U_c(c_t, t)}{U_c(c_0, 0)} d_t\right]$$

$$= E_0\left[\sum_1^{\infty} \frac{a\delta^t e^{-ac_t}}{a e^{-ac_0}} d_t\right]$$

$$= E_0\left[\sum_1^{\infty} \delta^t e^{a(c_0 - c_t)} d_t\right]$$

$$P_0 = E_0\left[\sum_1^{\infty} \delta^t e^{a(d_0 - d_t)} d_t\right]$$

3. For the Lucas model with labor income, show that assumptions (6.25) and (6.26) lead to the pricing relationship (6.27) and (6.28).

$$P_t = E_t\left[\sum_1^{\infty} \delta^j \left(\frac{c^*_{t+j}}{c^*_t}\right)^{\gamma-1} d_{t+j}\right]$$

$$\ln\left(\frac{c_{t+j}}{c_t}\right) = j\mu_c + \sigma_c \sum_1^j \eta_{t+j}$$

$$\frac{P_t}{d_t} = E_t\left[\sum_1^{\infty} \delta^j \left(\frac{c^*_{t+j}}{c^*_t}\right)^{\gamma-1} \frac{d_{t+j}}{d_t}\right]$$

$$\ln\left(\frac{d_{t+j}}{d_t}\right) = j\mu_d + \sigma_d \sum_1^j \varepsilon_{t+j}$$

$$= E_t\left[\sum_1^{\infty} \delta^j e^{(\gamma-1)\ln\left(\frac{c_{t+j}}{c_t}\right) + \ln\left(\frac{d_{t+j}}{d_t}\right)}\right]$$

$$= E_t\left[\sum_1^{\infty} \delta^j e^{\underbrace{(\gamma-1)(j\mu_c + \sigma_c \sum_1^j \eta_{t+j}) + j\mu_d + \sigma_d \sum_1^j \varepsilon_{t+j}}_{= \, j[(\gamma-1)\mu_c + \mu_d] + \sum_1^j [(\gamma-1)\sigma_c \eta_{t+j} + \sigma_d \varepsilon_{t+j}]}}\right]$$

$$= \sum_1^{\infty} e^{j\ln\delta} \cdot e^{j[(\gamma-1)\mu_c + \mu_d + \frac{1}{2}(1-\gamma)^2\sigma_c^2 + \frac{1}{2}\sigma_d^2 - (1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \sum_{1}^{\infty} e^{j\left[\ln\delta + (\gamma-1)\mu_c + \mu_d + \frac{1}{2}(1-\gamma)^2\sigma_c^2 + \frac{1}{2}\sigma_d^2 - (1-\gamma)\sigma_c\sigma_d\rho\right]}$$

$$= \sum_{1}^{\infty} e^{j\left[\ln\delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}\left((1-\gamma)^2\sigma_c^2 + \sigma_d^2\right) - (1-\gamma)\sigma_c\sigma_d\rho\right]}$$

let $\alpha = \ln\delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}\left((1-\gamma)^2\sigma_c^2 + \sigma_d^2\right) - (1-\gamma)\sigma_c\sigma_d\rho$

$$= \sum_{1}^{\infty} e^{j\alpha}$$

$$= \frac{1}{1-e^{\alpha}} - 1$$

$$= \frac{1}{1-\delta e^{\alpha^*}} - 1$$

$$= \frac{\delta e^{\alpha^*}}{1-e^{\alpha^*}}$$

where $\alpha^* = -(1-\gamma)\mu_c + \mu_d + \frac{1}{2}\left((1-\gamma)^2\sigma_c^2 + \sigma_d^2\right) - (1-\gamma)\sigma_c\sigma_d\rho$

4. Consider a special case of the model of rational speculative bubbles discussed in this chapter. Assume that infinitely-lived investors are risk-neutral and that there is an asset paying a constant, one-period risk-free return of $R_f = \delta^{-1} > 1$. There is also an infinitely-lived risky asset with price p_t at date t. The risky asset is assumed to pay a dividend of d_t which is declared at date t and paid at the end of the period, date $t+1$. Consider the price $p_t = f_t + b_t$ where

$$f_t = \sum_{i=0}^{\infty} \frac{E_t\left[d_{t+i}\right]}{{R_f}^{i+1}} \tag{1}$$

and

$$b_{t+1} = \begin{cases} \frac{R_f}{q_t} b_t + e_{t+1} & \text{with probability } q_t \\ z_{t+1} & \text{with probability } 1-q_t \end{cases} \tag{2}$$

where $E_t\left[e_{t+1}\right] = E_t\left[z_{t+1}\right] = 0$ and where q_t is a random variable as of date $t-1$ but realized at date t and is uniformly distributed between 0 and 1.

4.a Show whether or not $p_t = f_t + b_t$ subject to the specifications in (1) and (2) is a valid solution for the price of the risky asset.

Check if (2) satisfy $E_t[b_{t+1}] = R_f b_t$

$$E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] q_t + (1-q_t) E_t[z_{t+1}]$$

$$= b_t R_f \quad \checkmark \text{ Sol}^n$$

4.b Suppose that p_t is the price of a barrel of oil. If $p_t \geq p_{solar}$, then solar energy, which is in perfectly elastic supply, becomes an economically efficient perfect substitute for oil. Can a rational speculative bubble exist for the price of oil? Explain why or why not.

$$\lim_{j \to \infty} E_t[b_{t+j}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & b_t < 0 \end{cases}$$

We cannot have negative bubble path, so we consider only when $b_t > 0$.

It can be seen from the equation, bubble component must increase infinitely for a bubble to exist.

However, it cannot be a rational expectation if oil price has an upper bound, as it the case of perfect substitute in perfectly elastic supply.

So, b_t cannot rise above $P_{solar} - P_t^*$ and, thus, the bubble path cannot exist.

4.c Suppose p_t is the price of a bond that matures at date $T < \infty$. In this context, the d_t for $t \leq T$ denotes the bond's coupon and principal payments. Can a rational speculative bubble exist for the price of this bond? Explain why or why not.

No, it cannot. At final period T, its price must be $P_T = d_T$, zero after T.

This cannot satisfy the equation and increase infinitely.

Therefore, bubble path does not exist and the only rational price is P_t^*.

5. Consider an endowment economy with representative agents who maximize the following objective function:

$$\max_{C_s, \{\omega_{is}\}, \forall s, i} E_t \left[\sum_{s=t}^{T} \delta^s u(C_s) \right]$$

where $T < \infty$. Explain why a rational speculative asset price bubble could not exist in such an economy.

At final date T, $p_T = f_T = d_T$.

From $p_t = f_t + b_t$, it will be the case when $b_t = 0$; thus, $b_t = 0$ is certain.

Then, the bubble process implies $E_{T-1}[b_T] = \delta^{-1} b_{T-1} = E_{T-1}[0]$, $b_{T-1} = 0$.

Therefore, for all $t < T-1$, $b_t = 0$.