

Group Homework 7 & 8

Semester 2/2021 EE320 Introductory mathematical economics

Due date: May 7th 2022 (before midnight /B.E. moodle).

1. *Theory of firm*

Suppose that production function is given by $Q = f(K, L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to “ r ” and “ w ”, respectively. Consider the following problems.

- a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0 . Derive the factor inputs demand.
- b) Confirm your result with the second order derivative test.
- c) State the condition under which demand for capital and labor are both strictly positive.
- d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

2. *Theory of consumer*

Consider a household with the utility function given by,

$$U(x, y) = [x^2 + y^2]^{\frac{1}{2}}$$

where x and y are two different consumption goods, i.e. good x and good y . Suppose that (i) the prices for each of the two consumption goods are p_x and p_y respectively, and (ii) household's income is equal to M . Consider the following problems

- a) Calculate the total differential of the utility function.
- b) Set up the constrained optimization problem and derive the Marshallian demand function.
- c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?
- d) How does the demand for good y respond to price of good x ?
- e) What is the numerical value of λ^* when $M = \$300$, $p_x = 1$, $p_y = 1$?
- f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

- a) Suppose that fixed cost is equal to \$240. Calculate the total and the average cost when $q = 9$ units.
- b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.
- c) Calculate the social welfare under the monopoly environment.
- d) Calculate the social welfare loss under the monopoly environment.

4. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is \$7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.
