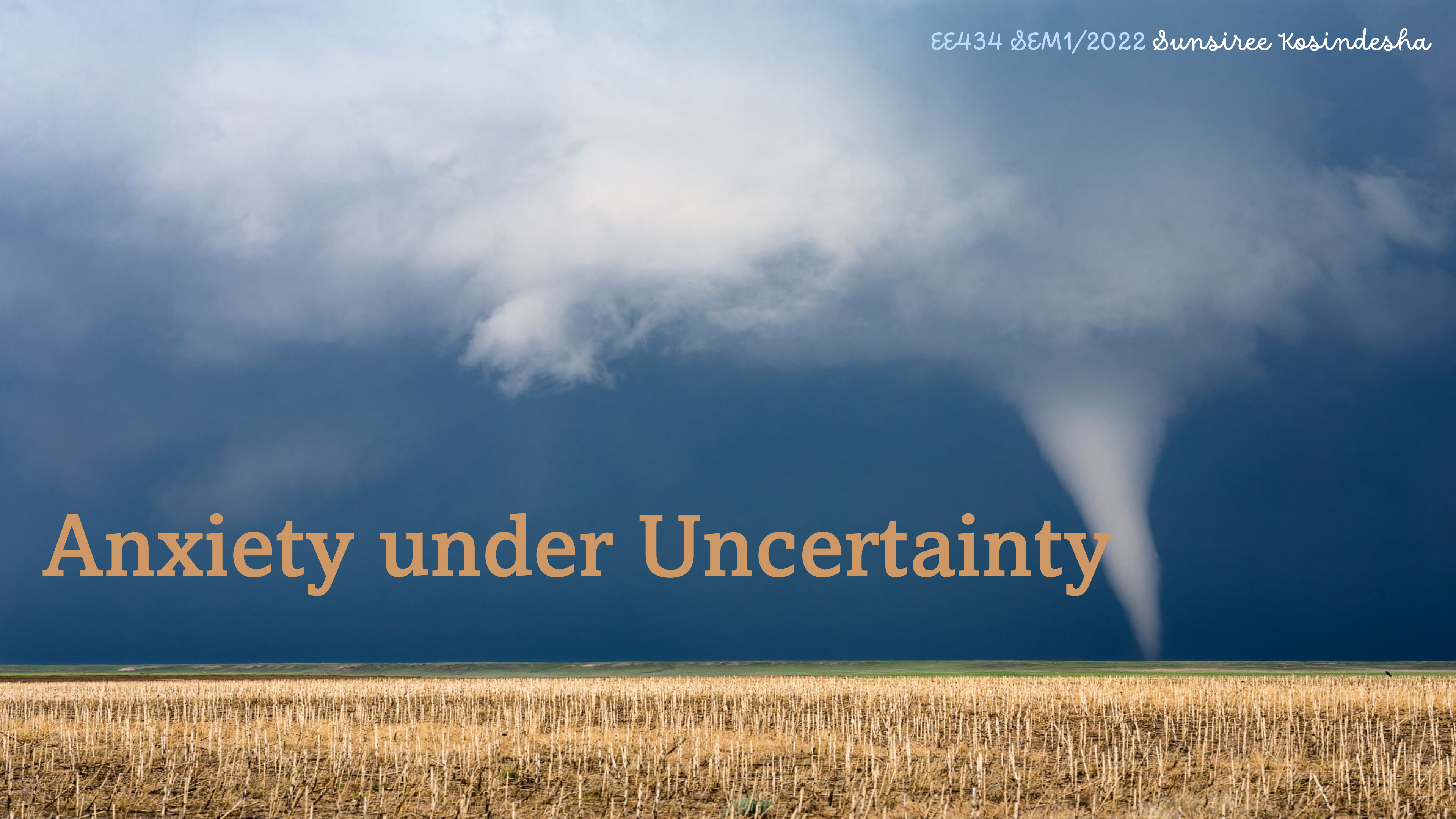

EE434 SEM1/2022 Sunsiree Kosindesha

Anxiety under Uncertainty

How do you feel about uncertainty?

- We all experience feelings related to our uncertainty about the future, such as hopefulness, anxiety, and suspense.
- These are “anticipatory emotions”.
- Anticipatory emotions are believed to arise in reaction to mental images of the outcome of a decision.

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

How do you feel about uncertainty?

- **Anxiety** is anticipatory and the desire to reduce anxiety motivates many decisions.

- The term anxiety denotes "**apprehension, tension, or uneasiness that stems from the anticipation of danger**".

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

Caplin&Leahy(2001)'s framework

Caplin and Leahy (2001) extends expected utility theory to situations in which agents experience feelings of anticipation prior to the resolution of uncertainty.

They model how lotteries over future physical prizes influence anticipatory emotions.

The model is called psychological expected utility.

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

Caplin&Leahy(2001)'s framework

- Caplin and Leahy (2001) also study the portfolio decisions of an anxious saver.
- The incorporation of anxiety into asset pricing models may help explain the equity premium puzzle and the risk-free rate puzzle.

Andrew Caplin, John Leahy, Psychological Expected Utility Theory and Anticipatory Feelings, *The Quarterly Journal of Economics*, Volume 116, Issue 1, February 2001, Pages 55–79, https://doi.org/10.1162/003355301556347

Caplin&Leahy(2001)'s framework

- Safe assets, by providing secure returns, may reduce anxiety even before final consumption takes place.

- Safe assets therefore provide an extra benefit in addition to the smoothing of final consumption across states. Thus, the risk-free rate can be reduced.

Caplin&Leahy(2001)'s framework

- Stocks and other risky assets, however, by increasing the variance of the portfolio, tend to increase anxiety in the period before final consumption takes place.
- Hence owning stocks involves an extra cost in addition to increasing the variance of final consumption, which increases their required return.

Anxiety vs. Standard Risk Aversion

- Anxious agent can appear to be risk averse, but the two concepts are different.
- Anxiety is emotion, related to the feeling of living with uncertainty.
- In contrast, “standard risk aversion” is a static concept pertaining to the curvature of the utility function within a period.

Anxiety vs. Risk aversion

- Anxiety complements risk aversion (and other explanations) in the understanding of the equity premium puzzle.
- In addition, anxious agents may appear to "overreact" to small probability events.

The General Framework

Caplin&Leahy(2001)'s framework

- Assume there are two periods: period $t = 1{,}2$.

- The feelings of anticipation concern the second-period uncertainty that remains unresolved after the outcome of the first-period lottery has been realized.

Caplin&Leahy(2001)'s framework

- Let X_1 represent the possible psychological states in period 1.
- Let X_2 represent the possible psychological states in period 2.
- Let Z_1 be the space of physical prizes in period 1.
- Let L_2 be the space of probability distributions over period 2 prizes.
- Define $Y_1 = Z_1 \times L_2$
- Define the function $\phi: Y_1 \to X_1$, giving the psychological state resulting from an agent facing the outcome $y_1 \in Y_1$.

Utility function

- Given $y_1 = (z_1, l_2) \in Y_1$,

$$V_1(y_1) = u_1\big(\phi(y_1)\big) + E_{l_2}[u_2(x_2)]$$

- V_1 looks like a standard time-separable expected utility function except for having the psychological states as argument.

Asset Prices and Anxiety

A model of portfolio choice

- Consider a two-period Lucas tree model of consumption and saving.

- A representative agent is born with an endowment of a consumption good equal to w_1. The consumption good is nonstorable.

For Lucas tree model, see Stokey, Nancy, and Robert Lucas, Recursive Methods in Economic Dynamics (Cambridge, MA: Harvard University Press, 1989), p.300.

A model of portfolio choice

- The agent is also endowed with N productive assets, each in fixed supply (normalized to unity), that yield random quantities of a consumption good in the second period.

- That is, a representative consumer is endowed with 1 unit each of N productive assets.

- Asset $\mathrm{n} \in \mathrm{N}$ yields s_{n} units of the consumption good in period 2.

A model of portfolio choice

- In addition to valuing consumption, the utility of the representative agent depends on the anxiety associated with holding risky assets.

- Let $Z_1 = Z_2 = R$ be the space of physical prizes in period 1 & 2.

- The consumption in each period is $c_t \in Z_t$.

- The space of temporal lotteries is the set of pairs

$$\mathrm{y}_1 = (\mathrm{c}_1, \mathrm{l}_2) \in \mathrm{Y}_1$$

, where l_2 represents a lottery over second-period consumption levels.

A model of portfolio choice

- The mapping $\phi: Y_1 \rightarrow X_1$ describes the relationship between temporal lotteries and the mental state in period 1.

- Assume that:

$$\phi(c_1, l_2) = (c_1, a(l_2))$$

, where $a: L_2 \rightarrow R$ is a differentiable function that measures the anxiety associated with the lottery $l_2 \in L_2$.

A model of portfolio choice

- The induced expected utility function takes the form

$$V_1(c_1, l_2) = u_1(c_1, a(l_2)) + \beta E_{l_2}[u_2(c_2)].$$

- l_2, a lottery over second-period consumption levels, depends on the portfolio that the agent holds.

- Let θ denote the vector of portfolio shares held by the agent, and θ_n the share of asset n.

- Define $\mathrm{l}_2(\theta)$ to be the distribution of the random variable $\sum_n s_n \theta_n$.

- Let $\tilde{a}(\theta)$ be the differentiable composite mapping $a(l_2(\theta))$, associating each portfolio to the corresponding level of anxiety.

A model of portfolio choice

- The agent chooses the level of first-period consumption and the asset portfolio to maximize utility subject to the budget constraint,

$$c_1 + \sum_n p_n \theta_n = w_1 + \sum_n p_n$$

$$c_2 = \sum_n s_n \theta_n$$

,where p_n is the price of asset n in terms of the consumption good in period 1.

A model of portfolio choice

- The first-order condition for asset n is:

$$\frac{\partial u_1}{\partial c_1} p_n = \frac{\partial u_1}{\partial a} \frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2}{\partial c_2} s_n\right].$$

The first-order condition for asset n

$$\frac{\partial u_1}{\partial c_1} p_n = \frac{\partial u_1}{\partial a}\frac{\partial \tilde{a}}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2}{\partial c_2} s_n\right].$$

- By reducing consumption by $\mathrm{p_n}$ units in the first period, the agent can purchase one unit of asset n, thereby raising consumption by s_{n} units in the second period.

- The portfolio adjustment, however, also has an effect on the level of anxiety in the first period reflected in the first term on the right-hand side of the FOC.

The competitive equilibrium

- First-period consumption is equal to the endowment:

$$c_1 = w_1$$

- Second-period consumption is equal to the random output of the assets:

$$c_2 = \sum s_n$$

Asset price

- Rearranging the FOC and plugging in the equilibrium conditions pins down the price of the asset:

$$p_n = \frac{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial a} \frac{\partial \tilde{a}(\mathbf{1})}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial c_1}}.$$

Asset price

$$p_n = \frac{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial a}\frac{\partial \tilde{a}(\mathbf{1})}{\partial \theta_n} + \beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2} s_n\right]}{\frac{\partial u_1(w_1, \tilde{a}(\mathbf{1}))}{\partial c_1}}.$$

- Since anxiety is aversive, $\frac{\partial \mathrm{u}_1}{\partial \mathrm{a}}$ is negative. It is immediate that an asset that causes anxiety has a lower price and a higher required rate of return.

Anxiety, Risk, and the Equity Premium

- Suppose that anxiety is decreasing in the mean and increasing in the riskiness of second-period consumption. Suppose that:

$$a(l_2) = -\alpha E_{l_2}(c_2) + \gamma var_{l_2}(c_2),\ \alpha, \gamma > 0$$

- Given that $c_2 = \sum s_n \theta_n$,

$$\frac{\partial\ \tilde{a}(\mathbf{1})}{\partial \theta_n} = -\alpha E_{l_2}(s_n) + 2\gamma cov_{l_2}(c_2, s_2)$$

Risk-free rate puzzle

- For a riskless asset, in which s_n is constant,

$$\frac{\partial\ \tilde{a}(\mathbf{1})}{\partial \theta_n} = -\alpha s_n < 0$$

Risk-free rate puzzle

- For a riskless asset, in which s_n is constant,

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- The price of the riskless asset is greater than the price it would take in the standard model.

Risk-free rate puzzle

- The agent is purchasing "peace of mind" along with the asset, and this justifies the low risk-free rate.

Equity premium puzzle

- Since stocks are risky, their purchase will tend to increase both the mean and the variance of second-period consumption.

- The sign of $\frac{\partial \, \tilde{a}(\mathbf{1})}{\partial \theta_n}$ will depend on how these two effects balance out. If γ is sufficiently large relative to α, the effect through the variance will dominate, and $\frac{\partial \, \tilde{a}(\mathbf{1})}{\partial \theta_n}$ will be positive.

Equity premium puzzle

- If $\frac{\partial\ \tilde{a}(\mathbf{1})}{\partial\theta_n} > 0$, then

$$p_n = \frac{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial a}\frac{\partial\tilde{a}(\mathbf{1})}{\partial\theta_n}+\beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2}s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}} < \frac{\beta E_{l_2}\left[\frac{\partial u_2(\sum_n s_n)}{\partial c_2}s_n\right]}{\frac{\partial u_1(w_1,\tilde{a}(\mathbf{1}))}{\partial c_1}}$$

- Anxiety will reduce the price of stocks and increase their return relative to the standard model.

Equity premium puzzle

- Stock ownership entails psychic costs. The agent has to live with the anxiety that accompanies the holding of a risky portfolio.

THANKS!

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