

EE311: Solution to the last problem set, Semester 1/2014

1. A monopolist firm faces a demand with constant elasticity of -2.0 . It has a constant marginal cost of \$20 per unit and sets a price to maximize profit. If marginal cost should increase by 25 percent, would the price charged also rise by 25 percent?

The monopolist's pricing rule is: $\frac{P - MC}{P} = -\frac{1}{E_d}$, or alternatively,

$$P = \frac{MC}{\left(1 + \left(\frac{1}{E_d}\right)\right)}. \text{ Therefore, price should be set so that } P = \frac{MC}{\left(1 + \frac{1}{-2}\right)} = 2MC.$$

With $MC = 20$, the optimal price is $P = 2(20) = \$40$. If MC increases by 25 percent to \$25, the new optimal price is $P = 2(25) = \$50$. So if marginal cost increases by 25 percent, the price also increases by 25 percent.

2. A firm has two factories for which costs are given by:

$$\text{Factory \#1: } C_1(Q_1) = 10Q_1^2$$

$$\text{Factory \#2: } C_2(Q_2) = 20Q_2^2$$

The firm faces the following demand curve:

$$P = 700 - 5Q$$

where Q is total output – i.e., $Q = Q_1 + Q_2$.

- a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing $Q = Q_1 + Q_2$). Indicate the profit-maximizing output for each factory, total output, and price.

The average revenue curve is the demand curve,

$$P = 700 - 5Q.$$

For a linear demand curve, the marginal revenue curve has the same intercept as the demand curve and a slope that is twice as steep:

$$MR = 700 - 10Q.$$

Next, determine the marginal cost of producing Q . To find the marginal cost of production in Factory 1, take the derivative of the cost function with respect to Q :

$$MC_1 = \frac{dC_1(Q_1)}{dQ_1} = 20Q_1.$$

Similarly, the marginal cost in Factory 2 is

$$MC_2 = \frac{dC_2(Q_2)}{dQ_2} = 40Q_2.$$

We know that total output should be divided between the two factories so that the marginal cost is the same in each factory. Let MC_T be this common marginal cost value. Then, rearranging the marginal cost equations in inverse form and horizontally summing them, we obtain total marginal cost, MC_T :

$$Q = Q_1 + Q_2 = \frac{MC_1}{20} + \frac{MC_2}{40} = \frac{3MC_T}{40}, \text{ or}$$

$$MC_T = \frac{40Q}{3}.$$

Profit maximization occurs where $MC_T = MR$. See the figure below for the profit-maximizing output for each factory, total output Q_T , and price.

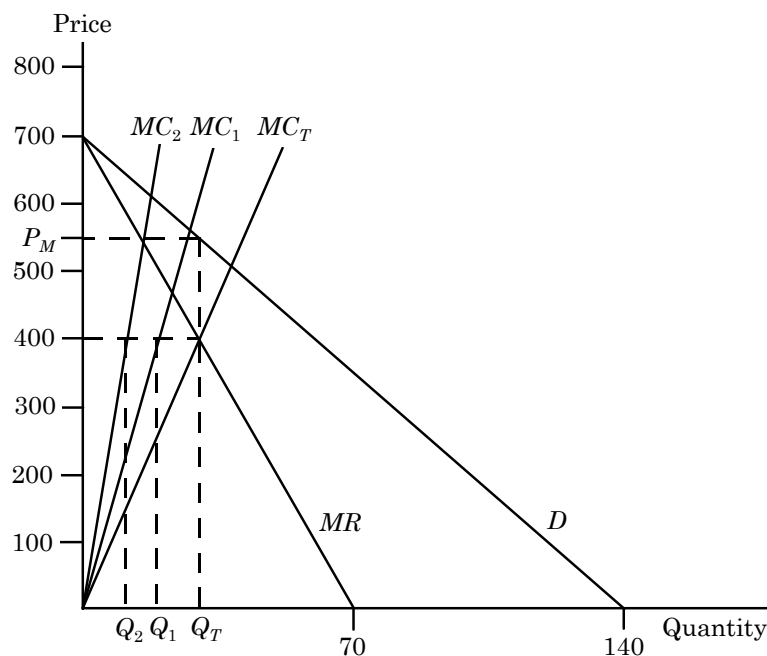


Figure 10.8.a

- b. Calculate the values of Q_1 , Q_2 , Q , and P that maximize profit.**

To calculate the total output Q that maximizes profit, set $MC_T = MR$:

$$\frac{40Q}{3} = 700 - 10Q, \text{ or } Q = 30.$$

When $Q = 30$, marginal revenue is $MR = 700 - (10)(30) = 400$. At the profit-maximizing point, $MR = MC_1 = MC_2$. Therefore,

$$MC_1 = 400 = 20Q_1, \text{ or } Q_1 = 20 \text{ and}$$

$$MC_2 = 400 = 40Q_2, \text{ or } Q_2 = 10.$$

To find the monopoly price, P , substitute for Q in the demand equation:

$$P = 700 - 5(30), \text{ or}$$

$$P_M = \$550.$$

- c. Suppose that labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust (i.e., raise, lower, or leave unchanged) the following: Output in Factory 1? Output in Factory 2? Total output? Price?**

An increase in labor costs will lead to a shift to the left in MC_1 , causing MC_T to shift to the left as well (since it is the horizontal sum of MC_1 and MC_2). The new MC_T curve will intersect the MR curve at a lower total quantity and higher marginal revenue. At a higher level of marginal revenue, Q_2 is greater than at the original level of MR . Since Q_T falls and Q_2 rises, Q_1 must fall. Since Q_T falls, price must rise.

3. A monopolist faces the demand curve $P = 11 - Q$, where P is measured in dollars per unit and Q in thousands of units. The monopolist has a constant average cost of \$6 per unit.
- a. Draw the average and marginal revenue curves and the average and marginal cost curves. What are the monopolist's profit-maximizing price and quantity? What is the resulting profit? Calculate the firm's degree of monopoly power using the Lerner index.

Because demand (average revenue) is $P = 11 - Q$, the marginal revenue function is $MR = 11 - 2Q$. Also, because average cost is constant, then marginal cost is constant and equal to average cost, so $MC = 6$.

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost:

$$11 - 2Q = 6, \text{ or } Q = 2.5.$$

That is, the profit-maximizing quantity equals 2500 units. Substitute the profit-maximizing quantity into the demand equation to determine the price:

$$P = 11 - 2.5 = \$8.50.$$

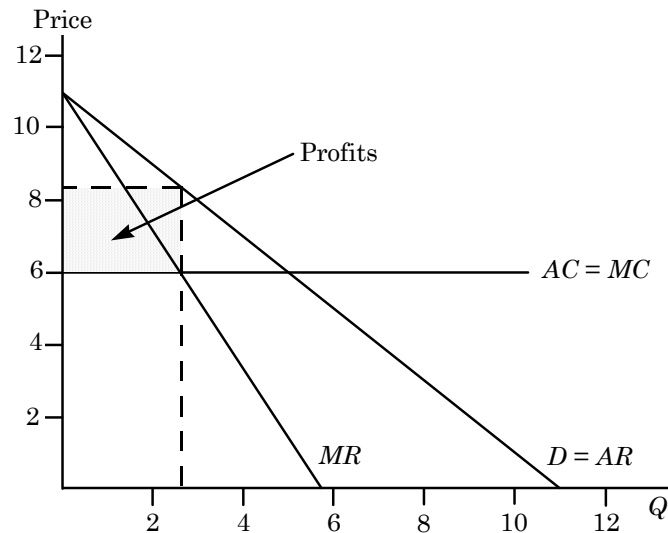
Profits are equal to total revenue minus total cost,

$$\begin{aligned} \pi &= TR - TC = PQ - (AC)(Q), \text{ or} \\ \pi &= (8.50)(2.5) - (6)(2.5) = 6.25, \text{ or } \$6250. \end{aligned}$$

The diagram below shows the demand, MR , AC and MC curves along with the optimal price and quantity and the firm's profits.

The degree of monopoly power according to the Lerner Index is:

$$\frac{P - MC}{P} = \frac{8.5 - 6}{8.5} = 0.294.$$



- b. A government regulatory agency sets a price ceiling of \$7 per unit. What quantity will be produced, and what will the firm's profit be? What happens to the degree of monopoly power?

To determine the effect of the price ceiling on the quantity produced, substitute the ceiling price into the demand equation.

$$7 = 11 - Q, \text{ or } Q = 4.$$

Therefore, the firm will choose to produce 4000 units rather than the 2500 units without the price ceiling. Also, the monopolist will choose to sell its product at the \$7 price ceiling because \$7 is the highest price that it can charge, and this price is still greater than the constant marginal cost of \$6, resulting in positive monopoly profit.

Profits are equal to total revenue minus total cost:

$$\pi = 7(4000) - 6(4000) = \$4000.$$

The degree of monopoly power falls to

$$\frac{P - MC}{P} = \frac{7 - 6}{7} = 0.143.$$

- c. What price ceiling yields the largest level of output? What is that level of output? What is the firm's degree of monopoly power at this price?**

If the regulatory authority sets a price below \$6, the monopolist would prefer to go out of business because it cannot cover its average variable costs. At any price above \$6, the monopolist would produce less than the 5000 units that would be produced in a competitive industry. Therefore, the regulatory agency should set a price ceiling of \$6, thus making the monopolist face a horizontal effective demand curve up to $Q = 5$ (i.e., 5000 units). To ensure a positive output (so that the monopolist is not indifferent between producing 5000 units and shutting down), the price ceiling should be set at $\$6 + \delta$, where δ is small.

4. As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues and fees for court time. There are two types of tennis players. “Serious” players have demand

$$Q_1 = 10 - P$$

where Q_1 is court hours per week and P is the fee per hour for each individual player. There are also “occasional” players with demand

$$Q_2 = 4 - 0.25P.$$

Assume that there are 1000 players of each type. Because you have plenty of courts, the marginal cost of court time is zero. You have fixed costs of \$10,000 per week. Serious and occasional players look alike, so you must charge them the same prices.

- a. Suppose that to maintain a “professional” atmosphere, you want to limit membership to serious players. How should you set the *annual* membership dues and court fees (assume 52 weeks per year) to maximize profits, keeping in mind the constraint that only serious players choose to join? What would profits be (per week)?

In order to limit membership to serious players, the club owner should charge an entry fee, T , equal to the total consumer surplus of serious players and a usage fee P equal to marginal cost of zero. With individual demands of $Q_1 = 10 - P$, individual consumer surplus is equal to:

$$(0.5)(10 - 0)(10 - 0) = \$50, \text{ or}$$

$$(50)(52) = \$2600 \text{ per year.}$$

An entry fee of \$2600 maximizes profits by capturing all consumer surplus. The profit-maximizing court fee is set to zero, because marginal cost is equal to zero. The entry fee of \$2600 is higher than the occasional players are willing to pay (higher than their consumer surplus at a court fee of zero); therefore, this strategy will limit membership to the serious players. Weekly profits would be

$$\pi = (50)(1000) - 10,000 = \$40,000.$$

- b. A friend tells you that you could make greater profits by encouraging both types of players to join. Is your friend right? What annual dues and court fees would maximize weekly profits? What would these profits be?

When there are two classes of customers, serious and occasional players, the club owner maximizes profits by charging court fees above marginal cost and by setting the entry fee (annual dues) equal to the remaining consumer surplus of the consumer with the lesser demand, in this case, the occasional player. The entry fee, T , equals the consumer surplus remaining after the court fee P is assessed:

$$T = 0.5Q_2(16 - P), \text{ where}$$

$$Q_2 = 4 - 0.25P.$$

Therefore,

$$T = 0.5(4 - 0.25P)(16 - P) = 32 - 4P + 0.125P^2.$$

Total entry fees paid by all players would be

$$2000T = 2000(32 - 4P + 0.125P^2) = 64,000 - 8000P + 250P^2.$$

Revenues from court fees equal

$$P(1000Q_1 + 1000Q_2) = P[1000(10 - P) + 1000(4 - 0.25P)] = 14,000P - 1250P^2.$$

Therefore, total revenue from entry fees and court fees is

$$TR = 64,000 + 6000P - 1000P^2.$$

Marginal cost is zero, so we want to maximize total revenue. To do this, differentiate total revenue with respect to price and set the derivative to zero:

$$\frac{dTR}{dP} = 6000 - 2000P = 0.$$

Solving for the optimal court fee, $P = \$3.00$ per hour. Serious players will play $10 - 3 = 7$ hours per week, and occasional players will demand $4 - 0.25(3) = 3.25$ hours of court time per week. Total revenue is then $64,000 + 6000(3) - 1000(3)^2 = \$73,000$ per week. So profit is $\$73,000 - 10,000 = \$63,000$ per week, which is greater than the $\$40,000$ profit when only serious players become members. Therefore, your friend is right; it is more profitable to encourage both types of players to join.

- c. Suppose that over the years young, upwardly mobile professionals move to your community, all of whom are serious players. You believe there are now 3000 serious players and 1000 occasional players. Would it still be profitable to cater to the occasional player? What would be the profit-maximizing annual dues and court fees? What would profits be per week?**

An entry fee of \$50 per week would attract only serious players. With 3,000 serious players, total revenues would be \$150,000 and profits would be \$140,000 per week. With both serious and occasional players, we may follow the same procedure as in part b. Entry fees would be equal to 4,000 times the consumer surplus of the occasional player:

$$T = 4000(32 - 4P + 0.125P^2) = 128,000 - 16,000P + 500P^2$$

Court fees are

$$P(3000Q_1 + 1000Q_2) = P[3000(10 - P) + 1000(4 - 0.25P)] = 34,000P - 3250P^2, \text{ and}$$

$$TR = 128,000 + 18,000P - 2750P^2.$$

$$\frac{dTR}{dP} = 18,000 - 5500P = 0, \text{ so } P = \$3.27 \text{ per hour.}$$

With a court fee of \$3.27 per hour, total revenue is $128,000 + 18,000(3.27) - 2750(3.27)^2 = \$157,455$ per week. Profit is $\$157,455 - 10,000 = \$147,455$ per week, which is more than the \$140,000 with serious players only. So you should set the entry fee and court fee to attract both types of players. The annual dues (i.e., the entry fee) should equal 52 times the weekly consumer surplus of the occasional player, which is $52[32 - 4(3.27) + 0.125(3.27)^2] = \1053 . The club's annual profit will be $52(147,455) = \$7.67$ million per year.