EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i		
				77.625
1	2.8	63	176.4	$(63-77.625)^2 = 213.890625$
2	3.4	72	244.8	$(72- \quad)^2 = 31.640625$
3	3	78	234	$(78- \quad)^2 = 0.140625$
4	3.5	81	283.5	$(81- \quad)^2 = 11.390625$
5	3.6	87	313.2	$(87- \quad)^2 = 87.890625$
6	3.0	75	225	$(75- \quad)^2 = 6.890625$
7	2.7	75	202.5	〃
8	3.7	90	333	$(90- \quad)^2 = 153.140625$
	25.7	621		511.875

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_0 = \bar{y} - \tilde{\beta}_1 \bar{x}$, $\tilde{\beta}_1 = \frac{\sum y_i x_i - n\bar{y}\bar{x}}{\sum(x_i - \bar{x})^2}$

$\sum y_i x_i = 2,012.4$

$\bar{y} = 3.2125$ $\quad \sum(x_i - \bar{x})^2 = 511.875$

$\bar{x} = 77.625$

$\tilde{\beta}_1 = \frac{2,012.4 - 8(3.2125)(77.625)}{511.875}$

$= 0.0340659$

$= 0.0341$ (3sf)

$\tilde{\beta}_0 = 3.2125 - 0.0340659(77.625)$

$= 0.5681318$

≈ 0.568 (3sf)

1.2 For each observation i, find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = 0.568 + 0.0341 x_i$

$\hat{y}_1 = 0.568 + 0.0341(63)$
$= 2.7142835 = 2.714$ (4dp)

$\hat{u}_1 = y_1 - \hat{y}_1 = 2.8 - 2.7142835$
$= 0.0857165$
$= 0.0857$ (4dp)

$\hat{y}_2 = 0.568 + 0.0341(72)$
$= 3.0208766 = 3.0209$ (4dp)

$\hat{u}_2 = 3.4 - 3.0208766 = 0.3791234$
$= 0.3791$ (4dp)

$\hat{y}_3 = 0.568 + 0.0341(78)$
$= 3.225272 = 3.2252$ (4dp)

$\hat{u}_3 = 3 - 3.225272 = 0.22572$
$= 0.2257$ (4dp)

$\hat{y}_4 = 0.568 + 0.0341(81)$
$= 3.3274697 = 3.3275$

$\hat{u}_4 = 3.5 - 3.327467$
$= 0.1725303 = 0.1725$

$\hat{y}_5 = 0.568 + 0.0341(87)$
$= 3.5318651 = 3.5319$

$\hat{u}_5 = 3.6 - 3.5318651$
$= 0.0681345 = 0.0681$

$\hat{y}_6 = 0.568 + 0.0341(75)$
$= 3.1230743 = 3.1231$

$\hat{u}_6 = 3.0 - 3.1230743$
$= -0.1230743$
$= -0.1231$

$\hat{y}_7 = 3.1230743 = 3.1231$ (4dp)

$\hat{u}_7 = 2.7 - 3.1230743$
$= -0.4230743 = -0.4231$ (4dp)

$\hat{y}_8 = 0.568 + 0.0431(90)$
$= 3.634065934 = 3.6341$ (4dp)

$\hat{u}_8 = 3.7 - 3.634065934$
$= 0.065934 = 0.0659$ (4dp)

$\sum_{i=0}^{N} \tilde{u}_i = 0$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.0857^2 + 0.3791^2 + 0.2257^2 + 0.1725^2 + 0.0681^2 + 0.1231^2 + 0.4231^2 + 0.0659^2}{6}$

$= 0.07245421245 = 0.0725$ (4dp)

$var(\hat{\beta}_0) = \frac{\sum X^2}{n \sum x^2} \sigma^2 = \frac{48717}{8(511.875)} \cdot (0.07245421245)$

$= 0.8619662681$

$= 0.8162$ (4dp)

$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum x^2} = \frac{0.07245421245}{511.875} = 0.0001415466$

$= 0.0001$ (4dp)

$n = 10$

2. Data is listed in the table

X_i	Y_i (91)	
10	0	0
12	2	24
14	5	70
16	6	96
18	7	126
22	10	220
24	10	240
26	15	390
28	16	448
30	20	600 = 2214

$(10 - 20)^2 = 100$
$(12 - 20)^2 = 64$
$(14 -)^2 = 36$
$(18 -)^2 = 4$
$(22 -)^2 = 4$
$(24 - 20)^2 = 16$
$(26 - 20)^2 = 36$
$(28 - 20)^2 = 64$
$= 100$
440

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$\hat{\beta}_0 = \bar{Y} - \tilde{\beta}_1 \bar{X}$, $\tilde{\beta}_1 = \frac{\sum y_i x_i - n\bar{y}\bar{x}}{\sum (X_i - \bar{X})^2}$

$\sum y_i x_i = 2214$
$\bar{y} = 9.1$
$\bar{x} = 20$
$\sum (X_i - \bar{X})^2 = 440$

$\tilde{\beta}_1 = \frac{2214 - 10(9.1)(20)}{440}$
$= 0.8954545455$
$= 0.8955$ (4dp)

$\hat{\beta}_0 = 9.1 - 0.8954545455(20)$
$= -8.809090909$
$= -8.8091$ (4dp)

The simple regression model $y_i = \beta_0 + \beta_1 x_i$ have a slope of 0.8955 ($\hat{\beta}_1$) and intercept the y-axis at -8.8091 ($\hat{\beta}_0$).

	X_i	Y_i
1	10	0
2	12	2
3	14	5
4	16	6
5	18	7
6	22	10
7	24	10
8	26	15
9	28	16
10	30	20

$\tilde{\beta}_1 = 0.8954545455$ A

$\tilde{\beta}_0 = -8.809090909$ B

2.2 Find the value of $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = -8.8091 + 0.8955 X_i$

$\hat{y}_1 = 0.1454545455 = 0.1455$ (4dp) $\quad \hat{u}_1 = -0.1454545455 = -0.1455$ (4dp)

$\hat{y}_2 = 1.936363636 = 1.9364$ (4dp) $\quad \hat{u}_2 = 0.06363636364 = 0.0636$ (4dp)

$\hat{y}_3 = 3.727272727 = 3.7273$ (4dp) $\quad \hat{u}_3 = 1.272727273 = 1.2727$ (4dp)

$\hat{y}_4 = 5.518181818 = 5.5182$ (4dp) $\quad \hat{u}_4 = 0.481818182 = 0.4818$ (4dp)

$\hat{y}_5 = 7.309090909 = 7.3091$ (4dp) $\quad \hat{u}_5 = -0.30909091 = -0.3091$ (4dp)

$\hat{y}_6 = 10.89090909 = 10.8909$ (4dp) $\quad \hat{u}_6 = -0.8909090909 = -0.8909$ (4dp)

$\hat{y}_7 = 12.68181818 = 12.6818$ (4dp) $\quad \hat{u}_7 = -2.681818182 = -2.6818$ (4dp)

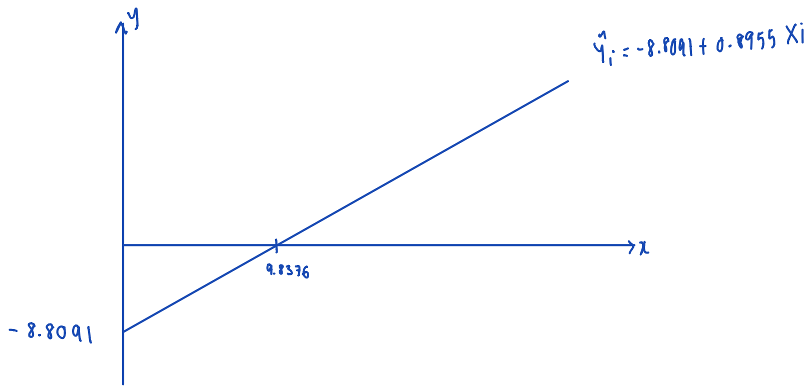
$\hat{y}_8 = 14.47272727 = 14.4727$ (4dp) $\quad \hat{u}_8 = 0.5272727273 = 0.5273$ (4dp)

$\hat{y}_9 = 16.26363636 = 16.2636$ (4dp) $\quad \hat{u}_9 = -0.2636363636 = -0.2636$ (4dp)

$\hat{y}_{10} = 18.05454545 = 18.0545$ $\quad \hat{u}_{10} = 1.945454545 = 1.9455$ (4dp)

$\sum \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



For $X_i = \bar{x} = 20$

$\hat{y}_i = -8.8091 + 0.8955(20)$

$= 9.1$

$= \bar{y}$

$\therefore$ the line passes $(\bar{x}, \bar{y})$

2.4 If $X_i = 16$, what is the predicted Y?

$\hat{y}_i = 5.518181818$

2.5 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{(-0.1455)^2 + (0.0636)^2 + (1.2727)^2 + (0.4818)^2 + (-0.3091)^2 + (-0.8909)^2 + (-2.6818)^2 + (0.5273)^2 + (-0.2636)^2 + (1.9455)^2}{8}$

note $x_i = X_i - \bar{X}$

$= 1.761726113 = 1.7617$ (4dp)

$var(\hat{\beta}_0) = \frac{\sum X^2}{n \sum x_i^2} \sigma^2 = \frac{4400}{10(440)} \, 1.761726113$

$= 1.828206344 = 1.761726113 = 1.7617$

$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.761726113}{440}$

$= 0.004003922$

$= 0.0040$ (4dp)

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

$$U_i = X_i - \bar{X} \qquad k_i = \frac{U_i}{\sum U_i^2}$$

$$\hat{\beta}_1 = \sum (Y_i - \bar{Y}) k_i$$

$$= \sum (\beta_0 + \beta_1 X_1 + u_i - \beta_0 - \beta_1 \bar{X}) k_i$$

$$= \beta_1 \sum (X_1 - \bar{X}) k_i + u_i k_i$$

$$= \beta_1 \sum U_i \cdot \frac{U_i}{\sum U_i^2} + u_i k_i$$

$$= \beta_1 + u_i k_i$$

$$E(\hat{\beta}_1) = E(\beta_1 + \sum U_i k_i)$$

$$= \beta_1 + E(\sum u_i k_i) \quad \rightarrow \text{SLR4}: E(u_i | X_i) = 0$$

$$= \beta_1 + \sum k_i E(u_i)$$

$$E(\tilde{\beta}_1) = \beta_1$$