EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\, u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \;\; and\; \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\; var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

Tharnnamrin Anghasuponkul
6104641037

1.

$$\bar{X} = \frac{63+72+78+81+87+75+75+90}{8} = \frac{621}{8} = 77.625$$

$$\bar{Y} = \frac{2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7}{8} = \frac{25.7}{8} = 3.2125$$

$$\sum_{i=1}^{n} x_i^2 = 63^2+72^2+78^2+81^2+87^2+75^2+75^2+90^2 = 48717$$

$$\sum_{i=1}^{n} x_i y_i = 63(2.8)+72(3.4)+78(3.0)+81(3.5)+87(3.6)+75(3.0)+75(2.7)+90(3.7)$$

$$= 2012.4$$

$$\sum_{i=1}^{n} x_i = 63+72+78+81+87+75+75+90 = 621$$

$$\sum_{i=1}^{n} y_i = 2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7 = 25.7$$

1.1

from
$$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$= \frac{8(2012.4) - 621(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641} = \frac{139.5}{4095} = 0.0341$$ #

from
$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 3.2125 - 0.0341(77.625)$$

$$= 0.5655$$ #

Tharnnumrin Anghasuponkul
6104641037

1.2

from $\hat{Y}_i = \beta_0 + \hat{\beta}_1 X_i$ where $\beta_0 = 0.5655$, $\beta_1 = 0.0341$

$$
\begin{aligned}
i=1, \quad \hat{Y}_1 &= 0.5655 + 0.0341(63) = 2.7138 \\
i=2, \quad \hat{Y}_2 &= 0.5655 + 0.0341(72) = 3.0207 \\
i=3, \quad \hat{Y}_3 &= 0.5655 + 0.0341(78) = 3.2253 \\
i=4, \quad \hat{Y}_4 &= 0.5655 + 0.0341(81) = 3.3276 \\
i=5, \quad \hat{Y}_5 &= 0.5655 + 0.0341(87) = 3.5322 \\
i=6, \quad \hat{Y}_6 &= 0.5655 + 0.0341(75) = 3.123 \\
i=7, \quad \hat{Y}_7 &= 0.5655 + 0.0341(75) = 3.123 \\
i=8, \quad \hat{Y}_8 &= 0.5655 + 0.0341(90) = 3.6345
\end{aligned}
$$

from $\hat{u}_i = Y_i - \hat{Y}_i$

$$
\begin{aligned}
i=1, \quad \hat{u}_1 &= 2.8 - 2.7138 = 0.0862 \\
i=2, \quad \hat{u}_2 &= 3.4 - 3.0207 = 0.3793 \\
i=3, \quad \hat{u}_3 &= 3.0 - 3.2253 = -0.2253 \\
i=4, \quad \hat{u}_4 &= 3.5 - 3.3276 = 0.1724 \\
i=5, \quad \hat{u}_5 &= 3.6 - 3.5322 = 0.0678 \\
i=6, \quad \hat{u}_6 &= 3.0 - 3.123 = -0.123 \\
i=7, \quad \hat{u}_7 &= 2.7 - 3.123 = -0.423 \\
i=8, \quad \hat{u}_8 &= 3.7 - 3.6345 = 0.0655
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n} \hat{u}_i &= 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.123 - 0.423 + 0.0655 \\
&= -0.0001 \approx 0
\end{aligned}
$$

Tharnnamrin Anghasuponkul
6104641037

1.3

from $\operatorname{var}(\hat{u}_i) = \dfrac{\sum \hat{u}_i^2}{n-2}$

$$= \frac{0.0862^2 + 3793^2 + (-0.2253)^2 + 0.1724^2 + 0.0674^2 + (-0.123)^2 + (-0.463)^2 + 0.0655^2}{8-2}$$

$$= \frac{0.4347}{6} = 0.0725 \ \#$$

from $\operatorname{var}(\hat{\beta}_0) = \dfrac{\sum X_i^2}{n \sum x_i^2} \sigma^2$

$$= \frac{48717}{8(514.3795)} (0.725)$$

$$= \frac{3531.9825}{4115.036} = 0.8583 \ \#$$

from $\operatorname{var}(\hat{\beta}_1) = \dfrac{\sigma^2}{\sum x_i^2} = \dfrac{\sigma^2}{\sum (X_i - \bar{X})^2}$

$$= \frac{0.0725}{\left[(63-77.625)^2 + (72-77.625)^2 + (78-77.625)^2 + (81-77.625)^2 + (87-77.625)^2 + (75-77.625)^2 + (75-77.625)^2 + (90-77.625)^2\right]}$$

$$= \frac{0.0725}{514.3795} = 0.00014 \ \#$$

Tharnnamrin Anghasuponkul

6104641037

2.

$$\bar{X} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{Y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

$$\Sigma x_i^2 = 10^2+12^2+14^2+16^2+18^2+22^2+24^2+26^2+28^2+30^2 = 4440$$

$$\Sigma x_i y_i = 10(0)+12(2)+14(5)+16(6)+18(7)+22(10)+24(10)+26(15)+28(16)+30(20)$$

$$= 2214$$

$$\Sigma x_i = 10+12+14+16+18+22+24+26+28+30 = 200$$

$$\Sigma Y_i = 0+2+5+6+7+10+10+15+16+20 = 91$$

2.1

from $\hat{\beta}_1 = \dfrac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2}$

$$= \frac{10(2214) - (200)(91)}{10(4440) - 200^2}$$

$$= \frac{3940}{4400} = 0.8955$$ → slope of regression line

from $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_1 \bar{X}$

$= 9.1 - 0.8955(20) = -8.81$ → Y-intercep of regression line, $\hat{Y}_0 = -8.81$

2.2

Tharnnumrin Anghusuponkul
6104641037

from $\hat{Y}_i = \beta_1 + \hat{\beta}_2 X_i$ where $\beta_1 = -8.81$, $\beta_2 = 0.8955$

$i=1,\quad \hat{Y}_1 = -8.81 + 0.8955(10) = 0.145$

$i=2,\quad \hat{Y}_2 = -8.81 + 0.8955(12) = 1.936$

$i=3,\quad \hat{Y}_3 = -8.81 + 0.8955(14) = 3.727$

$i=4,\quad \hat{Y}_4 = -8.81 + 0.8955(16) = 5.518$

$i=5,\quad \hat{Y}_5 = -8.81 + 0.8955(18) = 7.309$

$i=6,\quad \hat{Y}_6 = -8.81 + 0.8955(22) = 10.891$

$i=7,\quad \hat{Y}_7 = -8.81 + 0.8955(24) = 12.682$

$i=8,\quad \hat{Y}_8 = -8.81 + 0.8955(26) = 14.473$

$i=9,\quad \hat{Y}_9 = -8.81 + 0.8955(28) = 16.264$

$i=10,\quad \hat{Y}_{10} = -8.81 + 0.8955(30) = 18.055$

from $\hat{u}_i = Y_i - \hat{Y}_i$

$i=1,\quad \hat{u}_1 = 0 - 0.145 = -0.145$

$i=2,\quad \hat{u}_2 = 2 - 1.936 = 0.064$

$i=3,\quad \hat{u}_3 = 5 - 3.727 = 1.273$

$i=4,\quad \hat{u}_4 = 6 - 5.518 = 0.482$

$i=5,\quad \hat{u}_5 = 7 - 7.309 = -0.309$

$i=6,\quad \hat{u}_6 = 10 - 10.891 = -0.891$

$i=7,\quad \hat{u}_7 = 10 - 12.682 = -2.682$

$i=8,\quad \hat{u}_8 = 15 - 14.473 = 0.527$

$i=9,\quad \hat{u}_9 = 16 - 16.264 = -0.246$

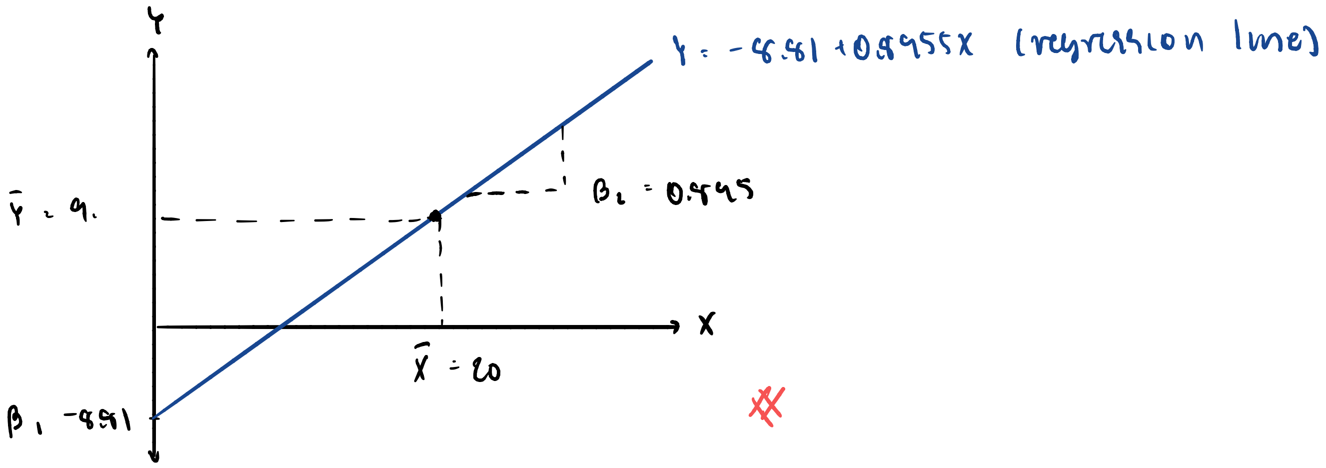
$i=10,\quad \hat{u}_{10} = 20 - 18.055 = 1.945$

$\Sigma \hat{u}_i = -0.145 + 0.064 + 1.273 + 0.482 - 0.309 - 0.891 - 2.682 + 0.527 - 0.246 + 1.945$

$= 0.018 \approx 0$

Thamnumrin Anghasuponkul
6104641037

2.3



Given $\bar{X} = 20, \bar{Y} = 9.1$ → substitute $\bar{X}$ into the regression function

$$Y = -8.81 + 0.8955(20)$$

$$= 9.1 = \bar{Y}$$

∴ the regression line passes $(\bar{X}, \bar{Y})$

2.4

from $\hat{Y}_i = \beta_1 + \beta_2 X_i$

$$\hat{Y}_i = -8.81 + 0.8955(18)$$

$$= 7.309$$

Tharnnamrin Anghasuponkul

6104641037

2.5

from $var(\hat{u}_i) = \dfrac{\Sigma \hat{u}_i^2}{n-2}$

$$= \frac{\left[(-0.145)^2 + 0.064^2 + 1.173^2 + 0.482^2 + (-0.309)^2 + (-0.8911)^2 + (-2.682)^2 + 0.517^2 + (-0.246)^2 + 1.945^2\right]}{10-2}$$

$$= \frac{14.08173}{8} = 1.7602$$

from $var(\hat{\beta}_1) = \dfrac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2$

$$= \frac{4440}{10(440)}(1.7602) = 1.7762$$

from $var(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{\sigma^2}{\Sigma (X_i - \bar{X})^2}$

$$= \frac{1.7602}{\left[(10-20)^2 + (12-20)^2 + (14-20)^2 + (16-20)^2 + (18-20)^2 + (22-20)^2 + (24-20)^2 + (26-20)^2 + (28-20)^2 + (30-20)^2\right]}$$

$$= \frac{1.7602}{440} = 0.00040$$

Tharnnamrin Anghasuponkul
6104641037

3.

from $\hat{\beta}_1 = \dfrac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2}$

$= \bar{Y} - \hat{\beta}_2 \bar{X}$

prove that this is an unbiased estimator

$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$

$= E(\bar{Y}) - \beta_2 E(\bar{X})$

$= \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2$

so $E(\hat{\beta}_1) = \beta_1 \rightarrow$ unbiased

\#

$\hat{\beta}_1$ is normally distributed with:

mean: $(\hat{\beta}_1) = \beta_1$