Solution: Quiz 3

1. Consider a function

$$f(x) = x^3 - 3x^2$$

   (a) Find the increasing and decreasing intervals.
   (b) Find the relative maximum and minimum.
   (c) Find the concavity and inflection point.
   (d) Sketch the graph of $f(x)$

Note: $f(x) = x^3 - 3x^2$, $f'(x) = 3x^2 - 6x$, $f''(x) = 6x - 6$,

Solution:

(a) and (b)

$$f'(x) = 3x^2 - 6x = 3x(x-2) = 0 \Rightarrow$$

$x = 0, 2$ are the only critical numbers.

	$x \in (-\infty, 0)$	$x \in (0, 2)$	$x \in (2, \infty)$
Sign of $f'(x) = 3x(x-2)$	$+$	$-$	$+$
$3x(x-2)$	$(-)(-)$	$(+)(-)$	$(+)(+)$

- From the table in (iii), we see that
$f(x)$ is increasing ($f'(x) > 0$) on $(-\infty, 0) \cup (2, \infty)$ and
$f(x)$ is decreasing ($f'(x) < 0$) on $(0, 2)$.

- The First Derivative Test (from the table above):
$f(0) = 0^3 - 30^2 = 0$ is a relative maximum
$f(2) = 2^3 - 3(2^2) = 2^2(2-3) = -4$ is a relative minimum.

(c) To find the inflection points, we first solve $f''(x) = 0$ and see the sign change of $f''(x)$:

$$f''(x) = 6x - 6 = 6(x-1) = 0 \qquad \Longrightarrow \qquad x = 1.$$

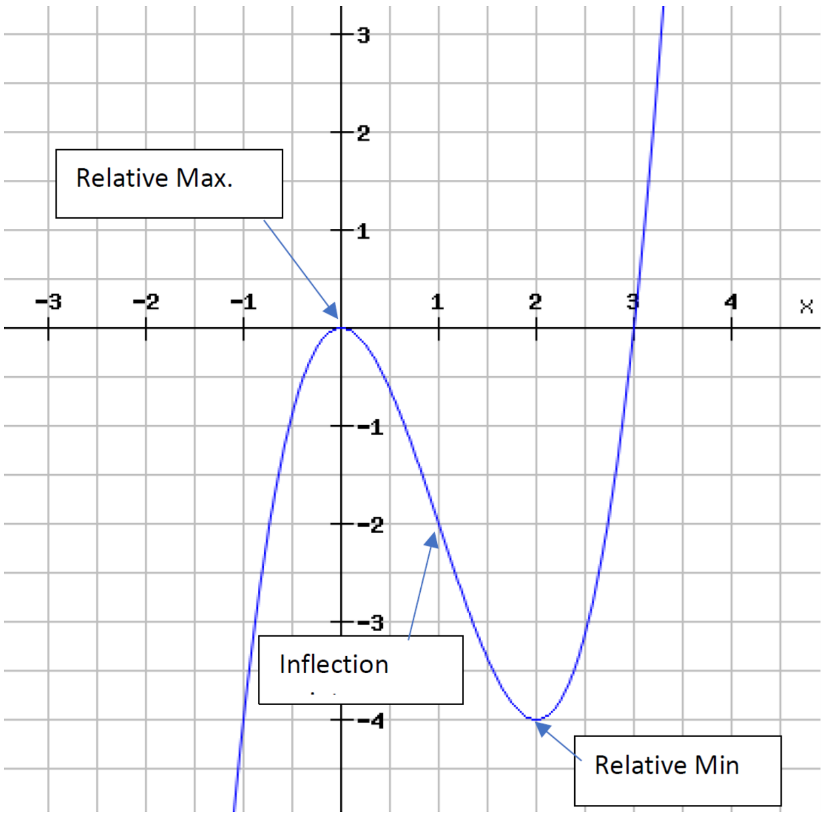
	$x \in (-\infty, 1)$	$x \in (1, \infty)$
Sign of $f''(x) = 6(x-1)$	$-$	$+$

Since the sign of $f''(x)$ changes at $x = 1$, it gives inflection points. Since $f(1) = 1^3 - 3(1^2) = -2$ the inflection points is $(1, -2)$.

From the table above, we see that
$f(x)$ is concave up ($f''(x) > 0$) on $(1, \infty)$ and
$f(x)$ is concave down ($f''(x) < 0$) on $(-\infty, 1)$.

(d)

(Next page)


Relative Max.
Inflection
Relative Min
x
-3
-2
-1
1
2
3
4
3
2
1
-1
-2
-3
-4