Group Homework 3 & 4

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 25th 2022 (before midnight /B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

b. Define the domain set of X. Is the function concave for all over the domain?

2. Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where *Q(L)* is the output level, and *L* is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).

b. What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?

c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (*MRP*) function.

3. Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 - 175Q + 10.$$

Consider the following questions.

a. Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?
b. Construct the profit function.
c. Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.
d. Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

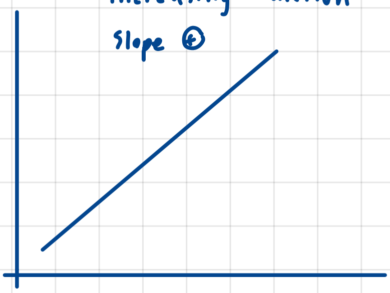
1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

b. Define the domain set of X. Is the function concave for all over the domain?

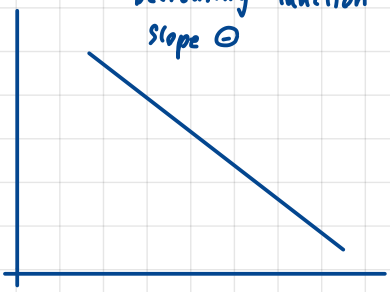
a) Increasing function

Slope ⊕



Decreasing function

Slope ⊖



Slope ⊕ $\frac{dy}{dx} > 0$

Slope ⊖ $\frac{dy}{dx} < 0$

$y = x^3 - 5x^2 + 7x - 5$

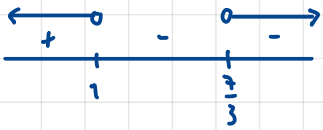
Find Slope $\rightarrow$ $\frac{dy}{dx} = 3x^2 - 10x + 7$

Increasing when $\frac{dy}{dx} > 0$

$3x^2 - 10x + 7 > 0$

$(3x - 7)(x - 1) > 0$

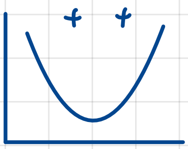
$x \rightarrow 1, \frac{7}{3}$

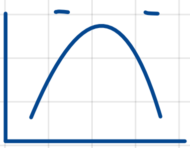


$\therefore$ Domain of X are $x < 1$ or $x > \frac{7}{3}$. #

$D(-\infty, 1) \cup (\frac{7}{3}, \infty)$

b)





convex

$\frac{d^2y}{dx} > 0$

concave

$\frac{d^2y}{dx} < 0$

$y = x^3 - 5x^2 + 7x - 5$

$\frac{dy}{dx} = 3x^2 - 10x + 7$

$\frac{d^2y}{dx^2} = 6x - 10$

concave when $\frac{d^2y}{dx^2} < 0$

$6x - 10 < 0$

$6x < 10$

$x < \frac{10}{6}$ #

$D(-\infty, \frac{10}{6})$

Yes, concave all over the domain.

2. Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where $Q(L)$ is the output level, and L is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).

b. What size of the work force (L^{**}) maximizes the average output per labor, $Q(L)/L$?

c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (MRP) function.

a) Average Product of labor $= \dfrac{Q}{L}$

Marginal Product of labor $= \dfrac{dQ}{dL}$

$Q = 6L^2 - L^3$

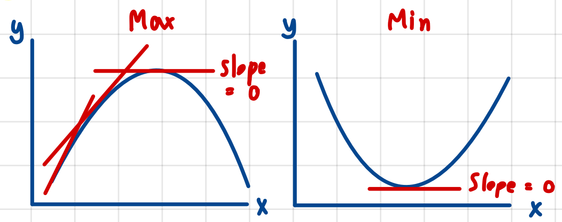
$AP_L = \dfrac{Q}{L} = \dfrac{6L^2 - L^3}{L} = 6L - L^2$ #

$MP_L = \dfrac{dQ}{dL} = 12L - 3L^2$ #

$\therefore AP_L = 6L - L^2$

$MP_L = 12L - 3L^2$ #

b) Max / Min



Slope $= 0$, $\dfrac{\Delta y}{\Delta x} = 0$, $\dfrac{dy}{dx} = 0$

Find L that makes Average Product of Labor (APL) "Max"

$AP_L = 6L - L^2$

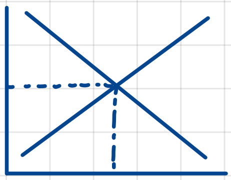
$\dfrac{dAP_L}{dL} = \boxed{6 - 2L = 0}$

$6 = 2L$

$L = 3$ People

$\therefore$ 3 labors of work force maximizes the average output per labor. #

c)



MP_L cross AP_L

$MP_L = AP_L$

from a)

$12L - 3L^2 = 6L - L^2$

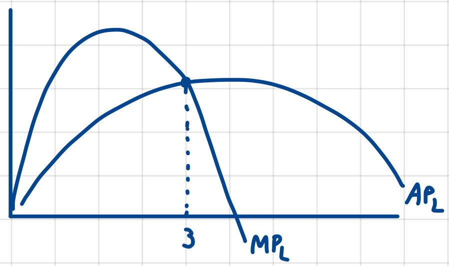
$0 = 2L^2 - 6L$

$0 = 2L(L-3)$

$0 = L(L-3)$

$L = 0, 3$

$\therefore L = 3$ same as b)



since MP_L cross AP_L at $L = 3$
same as Max AP_L at $L = 3$

d) $MRP = \dfrac{dTR}{dL} = MPL \cdot MR \rightarrow$ Marginal Revenue $= \dfrac{dTR}{dQ}$

$MP_L = 12L - 3L^2$

Find MR

Step 1 $\quad TR = P \cdot Q$

$Q = 100 - 2P$

$2P = 100 - Q$

$P = 50 - 0.5Q$

$TR = (50 - 0.5Q)Q$

$TR = 50Q - 0.5Q^2$

Step 2 $\quad MR = \dfrac{dTR}{dQ} = 50 - Q$

$MRP = MPL \cdot MR$

$= (12L - 3L^2)(50 - Q)$

change $Q \rightarrow L$

$Q(L) = 6L^2 - L^3$

$= (12L - 3L^2)(50 - 6L^2 - L^3)$ #

3. Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 - 175Q + 10.$$

Consider the following questions.

a. Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?

$TR = P \cdot Q$

$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right] Q$

$TR = 40Q + 105 - \frac{3}{2}Q^3$

Maximizing

$\frac{dTR}{dQ} : 40 - \frac{9}{2}Q^2 = 0$

$80 - 9Q^2 = 0$

$\frac{80}{9} = Q^2$

$2.98 = Q$

plug Q in Demand equation

$P = 40 + \frac{105}{2.98} - \frac{3}{2}(2.98)^2$

$= 61.91$

dQ/dP demand

$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$

$\frac{dQ}{dP} = \frac{1}{\frac{dP}{dQ}}$

$P = 40 + 105Q^{-1} - \frac{3}{2}Q^2$

$\frac{dP}{dQ} = 0 - 105Q^{-2} - 3Q$

so $\frac{dQ}{dP} = \frac{1}{-105Q^{-2} - 3Q}$

elasticity demand

$\frac{dQ}{dP} \cdot \frac{P}{Q}$

$= \frac{1}{-105Q^{-2} - 3Q} \cdot \frac{P}{Q}$ 2.98

$= \frac{1}{-105(2.98)^{2} - 3(2.98)} \cdot \frac{61.91}{2.98}$

$= -1$ #

b. Construct the profit function.

$$\text{Profit function} = TR - TC$$

$$= P \cdot Q - TC$$

$$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - 6\left[6Q^3 - 81Q^2 - 175Q - 10\right]$$

$$= -75Q^3 - 972Q^2 - 2020Q + 90$$

c. Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.

$$\pi = TR - TC$$

$$= PQ - TC$$

$$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - \left[6Q^3 - 81Q^2 - 175Q - 10\right]$$

$$= 40Q + 105 - \frac{3}{2}Q^3 - 6Q^3 + 81Q^2 + 175Q + 10$$

Maximize profit

$$\frac{d\pi}{dQ} = 40 + \frac{9}{2}Q^2 - 18Q^2 + 162Q + 175 = 0$$

$$= -13.5Q^2 + 162Q + 215 = 0$$

$$= 13.5Q^2 - 162Q - 215 = 0$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

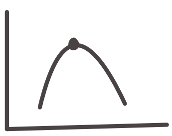
$$= \frac{-162 \pm \sqrt{(-162)^2 - 4(13.5)(-215)}}{2(13.5)}$$

$$= -1.21 \;,\; 13.21$$

Second order condition

$$\frac{d^2\pi}{dQ^2} = 9Q - 36Q + 162$$

$$= -27(13.21) + 162 \quad -$$



d. Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

$$\pi = TR - TC - [Tax]$$

$$\frac{d\pi}{dQ} = \frac{d}{dQ}[TR - TC - Tax] = 0 \quad \leftarrow \text{with tax}$$

$$\frac{dTR}{dQ} - \frac{dTC}{dQ} - \frac{dTax}{dQ} = 0$$

$$MR - MC - 0 = 0$$

$$MR = MC$$

$$\pi = TR - TC \quad \leftarrow \text{without tax}$$

$$\frac{d\pi}{dQ} = \frac{d}{dQ}[TR - TC] = 0$$

$$MR = MC$$

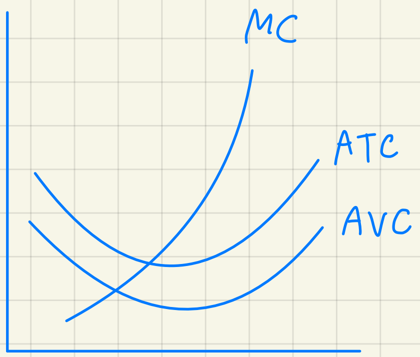
produce at MR = MC [same : not effective]

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

(a) Supply curve



(1) Find $MC = \frac{\Delta TC}{\Delta Q} = \frac{dTC}{dQ}$

$STC = 0.5Q^2 + 10Q + 5$

$MC = \frac{dTC}{dQ} = Q + 10$

$MC = Q + 10$

(2) Supply

$P = MC$

$P = Q + 10$

$Q = -10 + P$ → supply curve

(3) Market supply

$Q_M = Q_1 + Q_2 \ldots + Q_n$

$n = 100$

$Q_M = 100(P - 10)$

$Q^S_M = 100P - 1000$

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

(b) $Q_m^D = Q_m^S$

$1100 - 50P = -1000 + 100P$

$2100 = 150P$

$P = 14$

$Q_m = 400$

∴ Market Price is 14 THB and Market quantity 400 unit by 100 firms

So, Q of 1 firm $= \frac{400}{100} = 4$

Short-run profit

$\pi = P \cdot Q - TC$

$= 14(4) - (0.5(4)^2 + 10(4) + 5)$

$= 56 - 8 - 40 - 5$

$\pi = 3$ ✱

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

(c) tax ↑ = TC ↑

$STC = 0.5Q^2 + 10Q + 5 + 3Q$

$STC = 0.5Q^2 + 13Q + 5$

New supply

$MC = \frac{dSTC}{dQ} = Q + 13$

$MC = P$

$P = Q + 13$

$Q = -13 + P$ ← firm's supply

Market supply

$Q_m = 100(-13 + P)$

$Q_m = -1300 + 100P$ ✱

equilibrium $Q_D = Q_S$

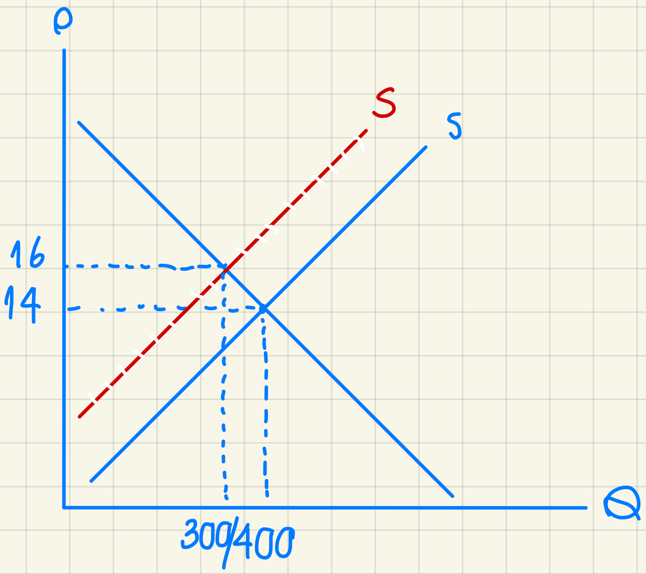
$1100 - 50P = -1300 + 100P$

$2400 = 150P$

$P = 16$

$Q = 300$ ✱

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?



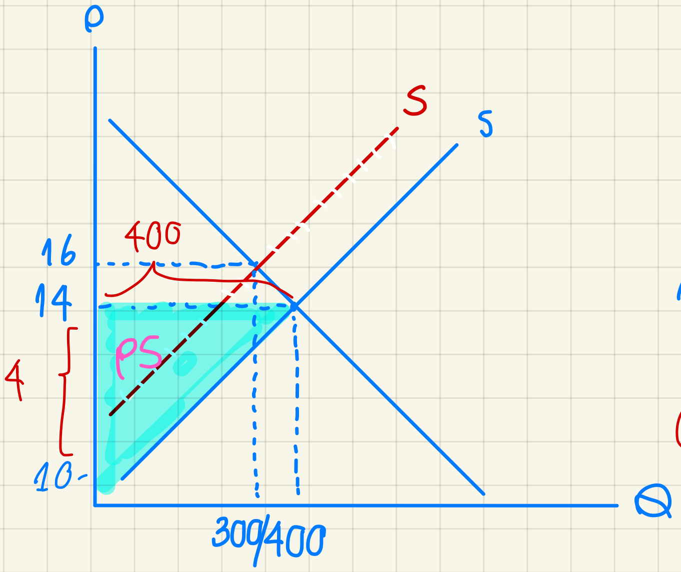
Consumer burden $= P^{n} - P^{o}$
$= 16 - 14$
$= 2$ THB/unit

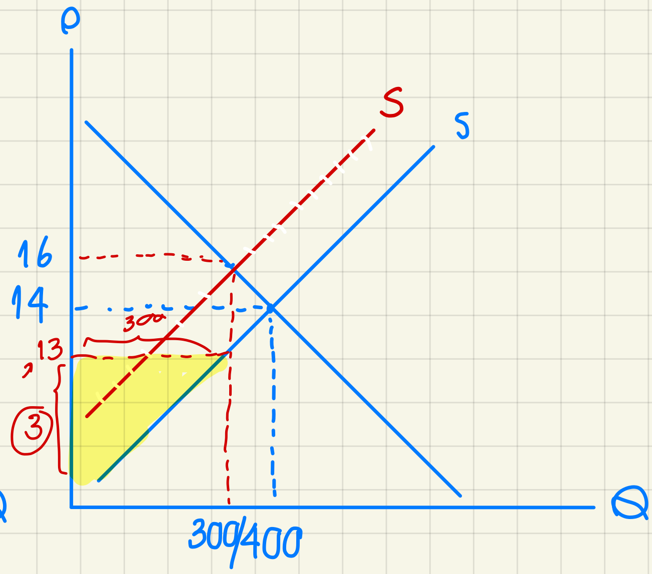
$\therefore \frac{2}{3_{Tax}} \times 100 = 66.67\%$

Producer burden $=$ tax $-$ consumer pay
$= 3 - 2$
$= 1$ THB/unit

$\therefore \frac{1}{3} \times 100 = 33.34\%$

e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.





$Q = -1000 + 100P$
$0 = -1000 + 100P$
$100P = 1000$
$P = 10$

$PS = 4 \times 400 \times \frac{1}{2}$
$= 800$

New PS

$\frac{1}{2} \times 3 \times 300 = 450$

Change in PS $= 450 - 800$
$= -350$

Change in total short-run profit

New profit

New Profit = $P \cdot Q - (TC + tax)$

$= 16(Q) - (0.5Q^2 + 10Q + 5 + 3Q)$

$q = \frac{300}{100} = 3$

$= 16(3) - (0.5(3^2) + 10(3) + 5 + 3(3))$

$= 48 - (4.5 + 30 + 5 + 9)$

$= 48 - 48.5$

$= -0.5$

Change in short run profit $= -0.5$

Total profit decrease $= -0.5(100)$

$= -50$