Mathematics for Economics

Chapter 0 Introduction

The objective of these lecture notes is to provide students with the foundation of mathematics that is most frequently used in economics. For a field as diverse as economics, it is impossible to cover even a fraction of all the mathematical topics an economics graduate students might encounter. Instead, the central aim will be the mathematics that are essential for any economist's tool box for the comparative static analyses: the optimization theory.

The lecture notes are divided into four parts: Linear algebra, Analysis, Calculus and Optimization. The first three parts do not only serve as the basis for the optimization theory, but also are themselves indispensable in modern economics. The exposition style adopted will be statements of definitions, lemmas, propositions, theorems and corollaries, followed by proofs. There are two reasons for this. The first is that this is the style prevalent in the contemporary economic literature and modern textbooks to cope with the complexity of recent theories, which in turn creates the need for the precision in the presentation of ideas. The second is that this is the only way to see the interwoven of mathematical concepts. The students are able to understand and then articulate why a statement is true and how it relates or depends on other statements previously proven to be true. This should be considered the real mathematical skills, not just knowing mechanically how to take partial derivatives and writing the optimality conditions. These skills will hopefully provide students with enough mathematical maturity to go out on their own to learn other branches of mathematics they might encounter in the future. They will find that, though the topics are different, the style of statements of definitions, lemmas, propositions, theorems and corollaries, followed by proofs will be the same.

For background preparation, the students should acquaint themselves with the three methods of proofs, i.e., deduction, induction, and contradiction, and the method of disproof by counterexample. See **Simon and Blume** [1994], Appendix A1.3, pages 851-858, and **Sydsaeter and Hammond** [1995], page 25-28. Writing proofs is a creative process and there are no systematic steps to follow. There are whole books written just on how to write proofs. See for example **Solow** [1990], volumes 1 and 2 of **Polya** [1954], and **Polya** [1945].

This lecture notes are intended to be used as the text for EE 621 Mathematical Economics in the MA programs at the Faculty of Economics, Thammasat University. Though a few economic examples are given, it has no intention to be a work about applications of mathematics in economics. The students are encouraged to read other books listed in the references for applications and also other alternative approaches and proofs. The selection of references is leaning toward those most readily available in the library and local bookstores.

3 Methods of Proofs.

1) Deduction — use any known math facts (result) to proceed step-by-step.

2) Induction

3) Contradiction

Ex. $\overbrace{\frac{2x}{2} = \frac{6}{2}}^{A} \Rightarrow \overbrace{x = 3}^{B}$.

x is a real number.

Induction — to prove a statement involving an integer n.

$A : \left(1+2+3+\cdots+n = \frac{n(n+1)}{2}\right)$

3 steps — 1). Prove the easiest case when $n = 1$.

$1 = \frac{1(1+1)}{2}$ ✓

2) State the Induction Hypothesis that the statement A is true ~~for~~ up to any integer (Theorem) up to n.

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$ ✓

3) Prove for the case of $n+1$

$\underbrace{1+2+\cdots+n}+(n+1) = \frac{n(n+1)}{2}+(n+1)$

$= (n+1)\left(\frac{n}{2}+1\right)$

$= \frac{(n+1)(n+2)}{2}$. ✓

H.W. ① $1^2+2^2+3^3+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$.

② $1^3+2^3+3^3+\cdots+n^3 = (1+2+3+\cdots+n)^2$

3) Contradiction. If we want to prove $\boxed{A} \to B$.

Ex $\sqrt{2}$ is not rational.

rational number is a ratio of 2 integers n, m

$$\sqrt{2} \neq \frac{n}{m}.$$

$$A \to B \iff \sim B \Rightarrow \sim A.$$

Cannot Prove by giving an example.

$n = 5$, 1000

But you can disprove a statement by showing an example.