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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i) = 5a + b\sum_{i=1}^{5} x_i$

   $= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

   b. $\sum_{y=0}^{5} f(x+y) = f(x+0) + f(x+1) + \ldots + f(x+5)$

   c. $\sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)[2(10)+1]}{6}$

   $= 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = \sum_{x=1}^{2}[2x+2+3]$

   $= \sum_{x=1}^{2} 2x+5 = 2(1)+2(2)+5 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b -0.0625	b -0.125	2.25b 0.28125	2b 0.25	1.5b 0.1875	0.5b 0.0625	0.25b 0.03125

** when b is constant number

   a. Find the value of b

   $f(x) = P(X=x) = 0.5b + b + 2b + 1.5b + 0.5b + 2.25b = 1$

   $8b = 1$

   $b = \frac{1}{8}$ #

   b. Find the answer for $P(X \le 2)$

   $= 1 - P(X > 2)$

   $= 1 - [P(X=3) + P(X=4)]$

   $= 1 - [0.0625 + 0.03125]$

   $= 1 - 0.09375$

   $= 0.90625$ #

   c. Find the answer for $P(-2 \le X \le 3)$

   $= P(X=-2) + P(X=-1) + P(X=0) + P(X=1) + P(X=2) + P(X=3)$

   $= -0.0625 - 0.125 + 0.28125 + 0.25 + 0.1875 + 0.0625$

   $= 0.59375$

   d. Find the answer for $P(X \ge 1)$

   $P(X \geq 1) = 1 - P(X < 1)$

   $= 1 - [P(X=0) + P(X=-1) + P(X=-2)]$

   $= 1 - (0.28125 - 0.125 - 0.0625)$

   $= 1 - 0.6875$

   $= 0.3125$

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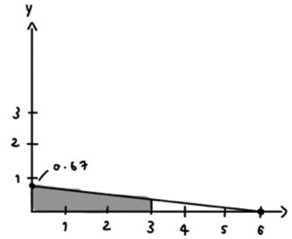
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



x	y
0	0.67
1	0.56
2	0.44
3	0.33

b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \leqslant x \leqslant 3) = \int_1^3 f(x)\,dx$$

$$= \left[-\frac{1}{18}x^2 + \frac{6}{9}x\right]_1^3$$

$$= \left[\frac{-(3)^2}{18} + \frac{6(3)}{9}\right] - \left[\frac{-(1)^2}{18} + \frac{6(1)}{9}\right] = \frac{-9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9} = \frac{16}{18}$$

c. Find the answer for $P(X \ge 2)$

$$P(X \geqslant 2) = \int_2^3 f(x)\,dx$$

$$= \left[\frac{-1}{18}x^2 + \frac{6}{9}x\right]_2^3$$

$$= \left[\frac{-1}{18}(3)^2 + \frac{6}{9}(3)\right] - \left[\frac{-1}{18}(2)^2 + \frac{6}{9}(2)\right] = \frac{7}{18}$$

d. Find the expected value of X

$$f(x) = -\frac{1}{9}x + \frac{6}{9}$$

$$E(X) = \int_{-\infty}^{\infty} x\,f(x)\,dx$$

$$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x\right)dx$$

$$= \left[\frac{-2}{9}x + \frac{6}{9}\right]_0^3$$

$$= \left[-\frac{2}{9}(3) + \frac{6}{9}\right] - \left[\frac{-2}{9}(0) + \frac{6}{9}\right]$$

$$= 2 \; \#$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

X \ Y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

   b. Find the marginal probability distribution function (PDF) of X

$P(X=0),\ P(X=1) = \frac{1}{2}$

   c. Find the marginal probability distribution function (PDF) of Y

$P(Y=1),\ P(Y=2),\ \dots,\ P(Y=6) = \frac{1}{6}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X \mid Y=1) = \sum_i x_i\, P(X=x_i \mid Y=1)$$

$$= \sum_i x_i \frac{P(X=x_i,\ Y=1)}{P(Y=1)} = \frac{1}{\frac{1}{6}}\left(\frac{1}{12}+\frac{1}{12}\right) = \cancel{6}^{1}\left(\frac{2}{\cancel{12}_{2}}\right) = 1$$

   e. Find the expected value of X given Y is equal to 1

$$E(X \mid Y=1) = \sum x_i\, P(X=x_i \mid Y=1) = \sum x_i\, P(X=x_i,\ Y=1) = \frac{1}{P(Y=1)} P(X=x_i, Y=1)$$

$$= \frac{1}{0.5}\left[\left(1\cdot\frac{1}{12}\right)+\left(2\cdot\frac{1}{12}\right)+\left(3\cdot\frac{1}{12}\right)+\left(4\cdot\frac{1}{12}\right)+\left(5\cdot\frac{1}{12}\right)+\left(6\cdot\frac{1}{12}\right)\right] = \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$Var(X \mid Y=1) = E(X - E(X \mid Y=1))^2 \cdot P(X \mid Y=1)$$

$$= \left[\left(1-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right) + \left(\left(2-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right) + \left(\left(3-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right) + \left(\left(4-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right) + \left(\left(5-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right) + \left(\left(6-\frac{7}{2}\right)^2\cdot\frac{1}{6}\right)$$

$$= \left(\frac{25}{4}\cdot\frac{1}{6}\right) + \left(\frac{9}{4}\cdot\frac{1}{6}\right) + \left(\frac{1}{4}\cdot\frac{1}{6}\right) + \left(\frac{1}{4}\cdot\frac{1}{6}\right) + \left(\frac{9}{4}\cdot\frac{1}{6}\right) + \left(\frac{25}{4}\cdot\frac{1}{6}\right)$$

$$= \frac{120}{24}$$

$$= \frac{10}{3}\ \#$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$

$$= \frac{1}{N} E(x_1 + x_2 + x_3)$$

$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3)\right]$$

$$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{3} \cdot 3\mu_x$$

$$= \mu_x$$

$$= \bar{x}$$

$$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$

$$= \frac{1}{N^2}\text{var}(x_1 + x_2 + x_3)$$

$$= \frac{1}{N^2}\left[\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + 2\,cov(x_1, x_2) + 2\,cov(x_1, x_3) + 2\,cov(x_2, x_3)\right]$$

$$= \frac{1}{3^2}\left[\frac{1}{4}\sigma^2_x + \frac{1}{4}\sigma^2_x + \frac{1}{4}\sigma^2_x + \frac{1}{2}\sigma^2_x + \frac{1}{2}\sigma^2_x\right]$$

$$= \frac{1}{3^2} \cdot \frac{9}{4}\sigma^2_x = \frac{\sigma^2}{4}x$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

   a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3) + E(x_4)\right]$$

$$= \frac{1}{4}\left[\mu_x + \mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

$$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{N^2}\text{var}(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{N^2}\left[\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + \text{var}(x_4)\right]$$

$$= \frac{1}{4^2}\left[\sigma^2_x + \sigma^2_x + \sigma^2_x + \sigma^2_x\right]$$

$$= \frac{1}{16}(4\sigma^2_x) = \frac{\sigma^2_x}{4} = 0.25\,\sigma^2_x$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$\tilde{X} = \frac{1}{4}(0.5x + x_2 + 0.5x_3 + 2x_4)$$

$$E(\tilde{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{4} E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$= \frac{1}{4}\left[E(0.5x_1) + E(x_2) + E(0.5x_3) + E(2x_4)\right]$$

$$= \frac{1}{4}\left[0.5E(x_1) + E(x_2) + 0.5E(x_3) + 2E(x_4)\right]$$

$$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

$\tilde{x}$ is unbiased estimator of μ

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{x}$ is a better estimator for μ than $\tilde{x}$ because it has smaller variance.