

1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^* .

$$Y = C + I + G$$

$$Y = a + b(Y - T) + I_0 + G_0$$

$$Y = a + bY - bT + I_0 + G_0$$

$$(1 - b)Y = a - bT + I_0 + G_0$$

$$Y^* = \frac{1}{1 - b} (a - bT + I_0 + G_0)$$

- Numerically, if $a = 1$, $T_0 = \$0$, $I_0 = \$1$, $G_0 = \$1$ and $b = 0.5$, this yields us,

$$Y^* = \frac{1}{1 - 0.5} (1 - 0.5(0) + 1 + 1)$$

$$Y^* = 2 \times 3$$

$$Y^* = 6$$

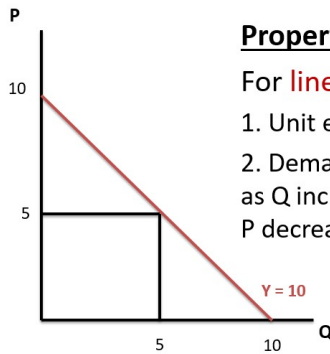
$$C^* = 1 + 0.5(6 - 0)$$

$$C^* = 4$$

$$Y_d^* = 6 - 0$$

$$Y_d^* = 6$$

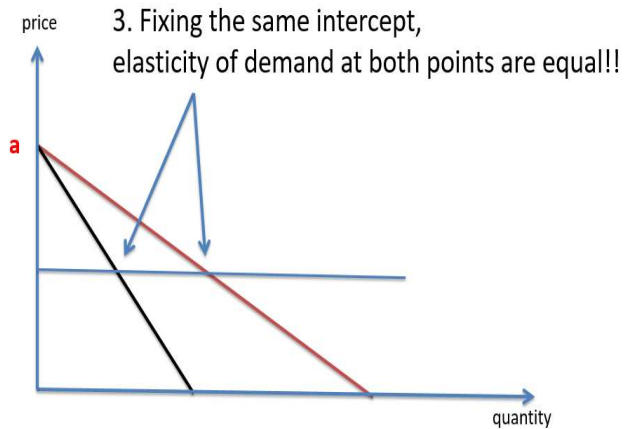
Linear function: slope v.s. elasticity



Property:

For **linear** demand curve:

1. Unit elasticity at the mid point.
2. Demand becomes more **inelastic** as Q increases, (correspondingly to P decreases)



use $\frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

- (i) elasticity of supply is always greater than 1 if $c > 0$,
- (ii) elasticity of supply is always equal to 1 if $c = 0$,
- (iii) elasticity of supply is always less than 1 if $c < 0$.

$$\begin{aligned}
 2.1) \quad PED &= \frac{dQ}{dP} \times \frac{P}{Q} \\
 &= -\frac{1}{b} \times \left(\frac{a-bQ}{Q} \right) \\
 &= \frac{-a+bQ}{bQ} \\
 &= 1 - \frac{a}{bQ}
 \end{aligned}$$

$$\begin{aligned}
 2.2) \quad PES &= \frac{1}{d} \times \left(\frac{c+dQ}{Q} \right) \\
 &= \frac{c+dQ}{dQ} \\
 &= 1 + \frac{c}{dQ} \\
 \text{if } c > 0, \quad PES &= 1 + \frac{c}{dQ} > 1 \\
 \text{if } c = 0, \quad PES &= 1 + \frac{c}{dQ} = 1 \\
 \text{if } c < 0, \quad PES &= 1 + \frac{c}{dQ} < 1
 \end{aligned}$$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

revenue function

$$TR(Q) = P(Q) \times Q$$

$$= (10 - Q) \times Q$$

$$= 10Q - Q^2$$

slope = $\frac{dTR}{dQ} = 10 - 2Q$

max occurs when $\frac{dTR}{dQ} = 0$

$$10 - 2Q = 0$$

$$Q = 5$$

At $Q = 5$, TR is max

$$TR = 25$$

- What is the break-even output?

$$TR = TC$$

$$4Q = 10Q - Q^2$$

$$Q^2 - 6Q = 0$$

$$Q = 0, 6$$

$$\therefore Q = 6$$

- What is the profit-maximizing level of output?

$$\pi = TR - TC$$

$$= 10Q - Q^2 - 4Q$$

$$\frac{d\pi}{dQ} = 10 - 2Q - 4 = 0$$

$$6 - 2Q = 0$$

$$Q = 3$$