

Assignment 2 Simultaneous Equations Model

Demand and Supply Equations

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t} \quad (2)$$

where: S_t = Domestic Supply at time t
 D_t = Domestic Demand at time t
 P_{Dt} = Domestic Price at time $t = P_{Mt} + T_t$
 T_t = Tariff at time t
 P_{X2t} = Price of Input 2 at time t
 P_{X3t} = Price of Input 3 at time t
 P_{X4t} = Price of Input 4 at time t
 GDP_t = Gross Domestic Product (Representing Income) at time t

Endogenous variables in this system include S_t , D_t , and P_{Dt}

Exogenous variables in this system include P_{X2t} , P_{X3t} , P_{X4t} , and GDP_t

From Data Assignment 2.dta:

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?
5. What do β_{21} and β_{22} mean?

1. State reduce form model of these system model

From Demand and Supply Equations,

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} \quad \text{--- ①}$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t} \quad \text{--- ②}$$

To find $\ln P_{Dt}$, Assume $\ln S_t = \ln D_t$

$$\beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

$$\beta_{11} \ln P_{Dt} - \beta_{21} \ln P_{Dt} = \beta_{20} + \beta_{22} \ln GDP_t + \varepsilon_{2t} - \beta_{10} - \beta_{12} \ln P_{X2t} - \beta_{13} \ln P_{X3t} - \beta_{14} \ln P_{X4t} - \varepsilon_{1t}$$

$$\beta_{11} \ln P_{Dt} - \beta_{21} \ln P_{Dt} = (\beta_{20} - \beta_{10}) - (\beta_{12} \ln P_{X2t}) - (\beta_{13} \ln P_{X3t}) - (\beta_{14} \ln P_{X4t}) + (\beta_{22} \ln GDP_t) + (\varepsilon_{2t} - \varepsilon_{1t})$$

$$(\beta_{11} - \beta_{21}) \ln P_{Dt} = (\beta_{20} - \beta_{10}) - (\beta_{12} \ln P_{X2t}) - (\beta_{13} \ln P_{X3t}) - (\beta_{14} \ln P_{X4t}) + (\beta_{22} \ln GDP_t) + (\varepsilon_{2t} - \varepsilon_{1t})$$

$$\ln P_{Dt} = \frac{(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{12} \ln P_{X2t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}}$$

$$\ln P_{Dt} = \frac{(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{12} \ln P_{X2t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} \quad \text{--- ③}$$

Plug $\ln P_{Dt}$ in $\ln S_t$

$$\ln S_t = \beta_{10} + \beta_{11} \left(\frac{(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{12} \ln P_{X2t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} \right) + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t}$$

$$= \beta_{10} + \frac{\beta_{11}(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{\beta_{11}\beta_{12} \ln P_{X2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{11}\beta_{13} \ln P_{X3t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{11}\beta_{14} \ln P_{X4t}}{\beta_{11} - \beta_{21}} + \frac{\beta_{11}\beta_{22} \ln GDP_t}{\beta_{11} - \beta_{21}} + \frac{\beta_{11}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t}$$

where

$$\pi_{10} = \beta_{10} + \frac{\beta_{11}(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} \quad \pi_{11} = \frac{-\beta_{11}\beta_{12}}{\beta_{11} - \beta_{21}} + \beta_{12} \quad \pi_{12} = \frac{-\beta_{11}\beta_{13}}{\beta_{11} - \beta_{21}} + \beta_{13}$$

$$\pi_{13} = \frac{-\beta_{11}\beta_{14}}{\beta_{11} - \beta_{21}} + \beta_{14} \quad \pi_{14} = \frac{\beta_{11}\beta_{22}}{\beta_{11} - \beta_{21}}$$

$$w_{1t} = \frac{\beta_{11}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \varepsilon_{1t}$$

$$\begin{aligned}
 \ln p_t &= \beta_{20} + \beta_{21} \left(\frac{(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{12} \ln p_{x2t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln p_{x3t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln p_{x4t})}{\beta_{11} - \beta_{21}} + \frac{\beta_{22} \ln GDP_t}{\beta_{11} - \beta_{21}} + \frac{(\xi_{2t} - \xi_{1t})}{\beta_{11} - \beta_{21}} \right) \\
 &\quad + \beta_{22} \ln GDP_t + \xi_{2t} \\
 &= \beta_{20} + \frac{\beta_{21}(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{\beta_{21}\beta_{12}(\ln p_{x2t})}{\beta_{11} - \beta_{21}} - \frac{\beta_{21}\beta_{13}(\ln p_{x3t})}{\beta_{11} - \beta_{21}} - \frac{\beta_{21}\beta_{14}(\ln p_{x4t})}{\beta_{11} - \beta_{21}} \\
 &\quad + \frac{\beta_{21}\beta_{22}(\ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{\beta_{21}(\xi_{2t} - \xi_{1t})}{\beta_{11} - \beta_{21}} + \beta_{22} \ln GDP_t + \xi_{2t}
 \end{aligned}$$

$$\begin{aligned}
 \text{where } \pi_{20} &= \beta_{20} + \frac{\beta_{21}(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} & \pi_{21} &= \frac{-\beta_{21}\beta_{12}}{\beta_{11} - \beta_{21}} & \pi_{22} &= \frac{-\beta_{21}\beta_{13}}{\beta_{11} - \beta_{21}} \\
 \pi_{23} &= \frac{-\beta_{21}\beta_{14}}{\beta_{11} - \beta_{21}} & \pi_{24} &= \frac{\beta_{21}\beta_{22}}{\beta_{11} - \beta_{21}} + \beta_{22} & w_{2t} &= \frac{\beta_{21}(\xi_{2t} - \xi_{1t})}{\beta_{11} - \beta_{21}} + \xi_{2t} \\
 \pi_{30} &= \frac{\beta_{20} - \beta_{10}}{\beta_{11} - \beta_{21}} & \pi_{31} &= \frac{-\beta_{12}}{\beta_{11} - \beta_{21}} & \pi_{32} &= \frac{-\beta_{13}}{\beta_{11} - \beta_{21}} \\
 \pi_{33} &= \frac{-\beta_{14}}{\beta_{11} - \beta_{21}} & \pi_{34} &= \frac{\beta_{22}}{\beta_{11} - \beta_{21}} & w_{3t} &= \frac{\xi_{2t} - \xi_{1t}}{\beta_{11} - \beta_{21}}
 \end{aligned}$$

The reduce form model of these system model

$$\begin{aligned}
 \ln s_t &= \pi_{10} + \pi_{11}(\ln p_{x2t}) + \pi_{12}(\ln p_{x3t}) + \pi_{13}(\ln p_{x4t}) + \pi_{14}(\ln GDP_t) + w_{1t} \\
 \ln d_t &= \pi_{20} + \pi_{21}(\ln p_{x2t}) + \pi_{22}(\ln p_{x3t}) + \pi_{23}(\ln p_{x4t}) + \pi_{24}(\ln GDP_t) + w_{2t} \\
 \ln p_t &= \pi_{30} + \pi_{31}(\ln p_{x2t}) + \pi_{32}(\ln p_{x3t}) + \pi_{33}(\ln p_{x4t}) + \pi_{34}(\ln GDP_t) + w_{3t}
 \end{aligned}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```

. use "/Users/pimpisam/Downloads/Assignment 2.dta"

. g pd = pm+t

. g lnst = ln(st)

. g lndt = ln(dt)

. g lnpd = ln(pd)

. g lnpx2 = ln(px2)

. g lnpx3 = ln(px3)

. g lnpx4 = ln(px4)

. g lngdp = ln(gdp)

. reg lnst lnpx2 lnpx3 lnpx4 lngdp

```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569724	4	1.16142431	F(4, 17)	=	37.32
Residual	.529104674	17	.031123804	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142	-.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442	-.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214	.5024549
lngdp	.3438812	.1913463	1.80	0.090	-.0598242	.7475865
_cons	24.65741	5.309757	4.64	0.000	13.4548	35.86002

. predict insthat
(option xb assumed; fitted values)

. reg lndt lnpx2 lnpx3 lnpx4 lngdp

Source	SS	df	MS	Number of obs	=	22
Model	3.4026552	4	.850663799	F(4, 17)	=	26.43
Residual	.54721789	17	.032189288	Prob > F	=	0.0000
				R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx2	-.4887365	.1541691	-3.17	0.006	-.8140049	-.1634682
lnpx3	-.7243134	.2830597	-2.56	0.020	-1.321517	-.1271097
lnpx4	-.577921	.4293997	-1.35	0.196	-1.483875	.3280333
lngdp	.1265855	.194594	0.65	0.524	-.2839719	.5371429
_cons	27.18614	5.399879	5.03	0.000	15.79339	38.57889

```
. predict lndhat
(option xb assumed; fitted values)
```

```
. reg lnpx2 lnpx3 lnpx4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	.17707359	4	.044268398	F(4, 17)	=	6.76
Residual	.111247189	17	.006543952	Prob > F	=	0.0019
				R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpx2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnpx3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnpx4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

```
. predict lnpdhat
(option xb assumed; fitted values)
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. reg lnst lnpdhat lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569773	4	1.16142443	F(4, 17)	=	37.32
Residual	.529104183	17	.031123775	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

. reg lndt lnpdhat lngdp

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
				R-squared	=	0.8257
				Adj R-squared	=	0.8073
Total	3.94987309	21	.188089195	Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?

. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls nodfk inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	13.81	0.0000
lndt	22	2	.1454858	0.8982	89.20	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	2.10611	1.926677	1.09	0.282	-1.801371	6.013591
lnpx2	-.7279628	.3026471	-2.41	0.021	-1.34176	-.114166
lnpx3	-1.122146	.464304	-2.42	0.021	-2.063798	-.180494
lnpx4	-1.428722	.7811544	-1.83	0.076	-3.012977	.1555325
_cons	18.59914	14.05113	1.32	0.194	-9.897873	47.09616
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.394875	-1.75344
lngdp	.5212921	.0955104	5.46	0.000	.327588	.7149961
_cons	35.93499	5.106302	7.04	0.000	25.57893	46.29105

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnp2 lnp3 lnp4) (lndt lnpd lngdp), 3sls inst(lnpx2 lnp3 lnp4)
> 4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnt lnpd lndt

Exogenous variables: lnp2 lnp3 lnp4 lngdp

```
. reg3 (lnst lnpd lnp2 lnp3 lnp4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnp
> 3 lnp4 lngdp)
```

```
Iteration 1:  tolerance =  .1059484
Iteration 2:  tolerance =  .04569793
Iteration 3:  tolerance =  .01846611
Iteration 4:  tolerance =  .00725496
Iteration 5:  tolerance =  .00281814
Iteration 6:  tolerance =  .00108981
Iteration 7:  tolerance =  .00042072
Iteration 8:  tolerance =  .00016231
Iteration 9:  tolerance =  .0000626
Iteration 10: tolerance =  .00002414
Iteration 11: tolerance =  9.310e-06
Iteration 12: tolerance =  3.590e-06
Iteration 13: tolerance =  1.384e-06
Iteration 14: tolerance =  5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnp2 lnp3 lnp4 lngdp

Concerning the asymptotic property, 2SLS is the most appropriate model because OLS cannot be used due to the endogeneity problem in the model (OLS would give us bias, inconsistent and inefficiency) and the 3SLS and I3SLS is not preferred to use since we do not know that there are the specification error in the model or not which might lead us to the inconsistent.

In 2SLS, using reduced form and instrumental variables (IV), the endogeneity problem is solved and it gives consistency and efficiency.

5. What do β_{21} and β_{22} mean?

β_{21} represents the price elasticity of demand. If domestic prices change by 1%, how many percent change in quantity of domestic demand.

β_{22} represents the income elasticity of demand. If GDP which represents the income increases by 1%, how many percent change in quantity of domestic demand.