B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2015

In-Class Exercise 3

1. Find the rate of change of output with respect to time (*dQ/dt*), if the production function is $Q = A(t)K^{\alpha}L^{\beta}$, where *A(t)* is an increasing function of *t*, and $K = K_0 + at$ and $L = L_0 + bt$.

Ans.

$$\frac{dQ}{dt} = Q_A\frac{dA}{dt} + Q_K\frac{dK}{dt} + Q_L\frac{dL}{dt}$$

$$\frac{dQ}{dt} = K^{\alpha}L^{\beta}A'(t) + \alpha AK^{\alpha-1}L^{\beta}.(a) + \beta AK^{\alpha}L^{\beta-1}.(b)$$

$$\frac{dQ}{dt} = K^{\alpha}L^{\beta}A'(t) + \alpha a\frac{AK^{\alpha}L^{\beta}}{K} + \beta b\frac{AK^{\alpha}L^{\beta}}{L} = \left[A'(t) + \frac{\alpha aA}{K} + \beta b\frac{\beta bA}{L}\right]K^{\alpha}L^{\beta}$$

2. Find the total differential, given

a. $U = -5x^3 - 12xy - 6y^5$

$dU = (-15x^2 - 12y)dx + (-12x - 30y^4)dy$

b. $z = (5x^2 + 7y)(2x - 4y^3)$

$z = 10x^3 - 20x^2y^3 + 14xy - 28y^4$

⇨ $dz = (30x^2 - 40xy^3 + 14y)dx + (-60x^2y^2 + 14x - 112y^3)dy$

Alternatively, let $u = 5x^2 + 7y$ and $v = 2x - 4y^3$.

$du = 10xdx + 7dy\ ;\ dv = 2dx - 12y^2dy$
$dz = u \cdot dv + v \cdot du$

⇨ $dz = (5x^2 + 7y)(2dx - 12y^2dy) + (2x - 4y^3)(10xdx + 7dy\ )$
$= (30x^2 - 40xy^3 + 14y)dx + (-60x^2y^2 + 14x - 112y^3)dy$

3. Derive $\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial z}$ from the following implicit functions.

a. $x^2y^3 + z^2 + xyz = 0$

Let $F(x, y) \equiv x^2y^3 + z^2 + xyz = 0$.

⇨ $\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = -\frac{y^3+yz}{3x^2y^2+xz}$

⇨ $\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = -\frac{2z+xy}{3x^2y^2+xz}$

b. $x^2y^2z + e^x - e^y + e^z = 0$

Let $F(x, y) \equiv x^2y^2z + e^x - e^y + e^z = 0$.

⇨ $\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = -\frac{2xy^2z+e^x}{2x^2yz-e^y}$

⇨ $\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = -\frac{x^2y^2+e^z}{2x^2yz-e^y}$