EE 320
Introductory Mathematical Economics
Semester 1/2011

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Homework # 4

Answer Key

1.

$10 = L^{3/8}K^{5/8}$

$$L^{3/8} = 10K^{-5/8}$$

or

$$L = 10^{8/3}K^{(-5/8)(8/3)} = 10^{8/3}K^{-5/3}$$

thus

$$dL/dK = 10^{8/3}\left(\frac{-5}{3}\right) K^{-8/3}$$

dL/dK gives the slope of the isoquant or a measure of the marginal rate of technical substitution.

Costs will be at a minimum when the firm's isocost function is tangential to this isoquant. Since $P_L = 3$ and $P_K = 5$, the isocost function will take the form 3L + 5K = N, where N stands for costs. Along the isocost line dL/dK is thus -5/3.

Costs are at a minimum when the slope of the tangent to the isoquant is equal to the slope of the isocost function, i.e. when

$$10^{8/3}\left(\frac{-5}{3}\right) K^{-8/3} = -5/3$$

or

$$K = 10$$

When K = 10, $10 = L^{3/8}\, 10^{5/8}$, or L= 10.

When K = L = 10, then 3L + 5K = 3(10) + 5(10) = 80.

Therefore the minimum level of costs is 80 and the algebraic form of the isocost function is 3L + 5K = 80.

3.

The budget constraint is 100 = 5x + 10y and so let L = 6xy + λ(100 – 5x – 10y).

FOC: $L_x = 6y + 5\lambda = 0$

$L_y = 6x + 10\lambda = 0$

$L_\lambda = 100 - 5x - 10y = 0$

x = 2y so that x = 10, y = 5 and λ = 6.

SOC:

$$\begin{vmatrix} 0 & -5 & -10 \\ -5 & 0 & 6 \\ -10 & 6 & 0 \end{vmatrix} = 600 > 0$$

$x = 10$, $y = 5$ gives a maximum with $U = 6xy = 300$.