ENDOGENOUS GROWTH MODELS

EE 462 Development Macroeconomics

Semester 1/2014

Topics

- Transitional Dynamics in Solow Model
- The AK Model
- Basic Human Capital Model
- The Lucas Model
- Growth Model with Externalities
- Romer's Model

Transitional Dynamics (Solow Model)

- Recall from Solow model, capital accumulation can be written as:

$$\Delta k = sy - (n+d)k$$

- Rewrite in terms of the rate of change in k:

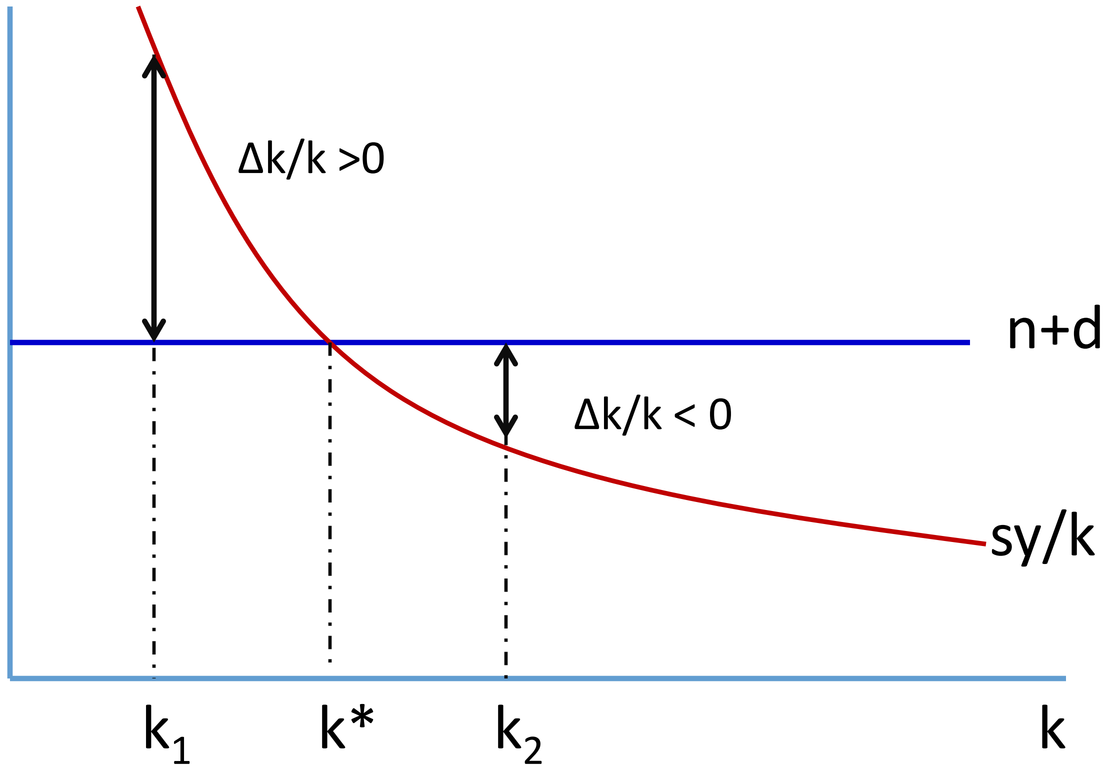
$$\Delta k/k = sy/k - (n+d)$$

- Given the production function: $y = f(k) = k^{\alpha}$. Then,

$$\rightarrow \quad \Delta k/k = sk^{\alpha-1} - (n+d)$$

  - Countries with high levels of *k* ($k > k_{ss}$) will have negative growth rates of *k*.
  - Countries with low levels of *k* ($k < k_{ss}$) will have a positive growth rates of *k*.

Graph: Transitional Dynamics in Solow Model



The AK Model

- Assume a production function without diminishing returns:

  $Y = AK$ where $A > 0$

- Output per capita can be written as:

  $y = Ak$

  where $y = Y/L$ and $k = K/L$

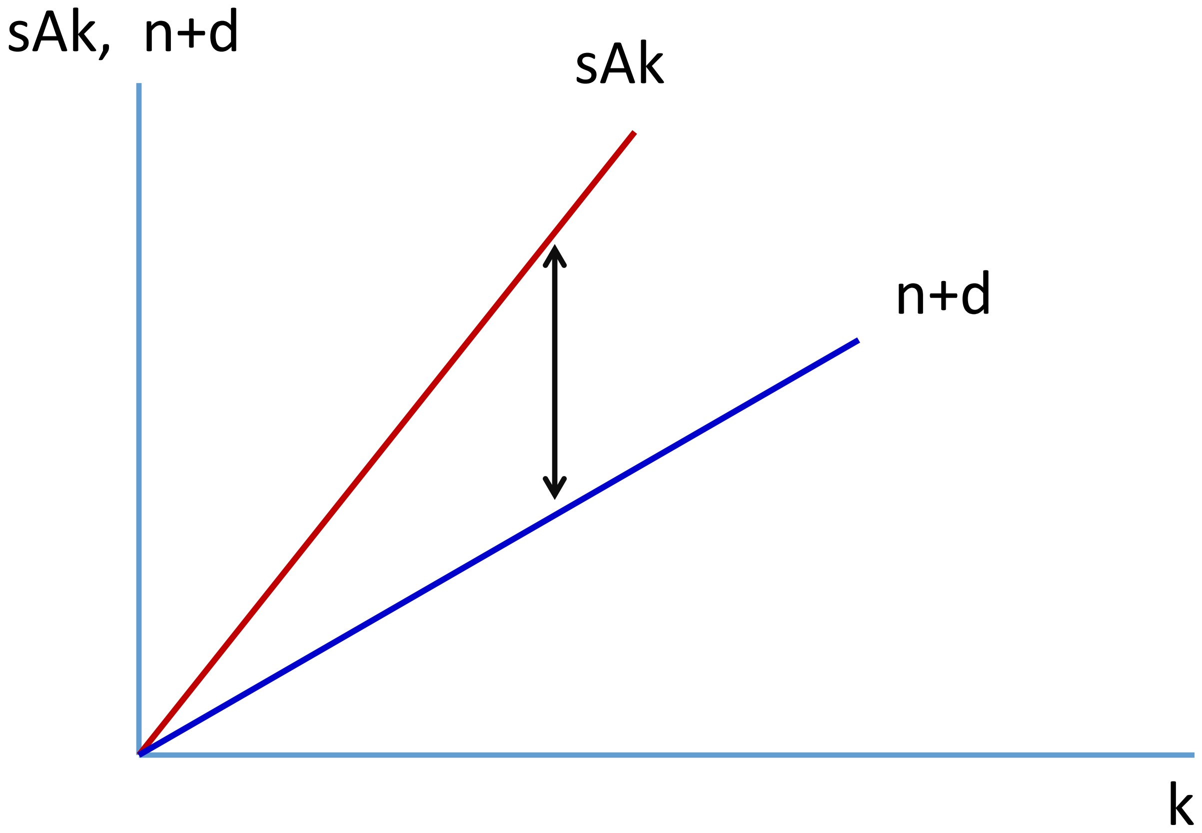
- The accumulation of capital per worker can be written as:

  ➔ $\Delta k/k = sA - (n+d)$

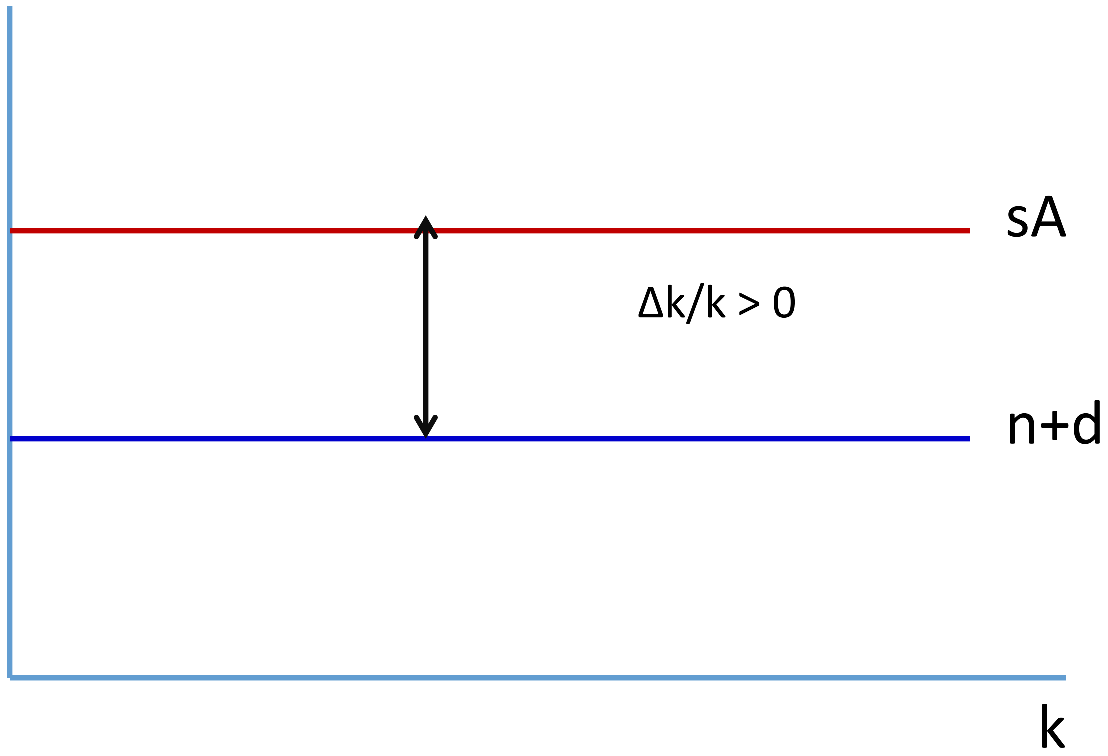
  ➔ $\Delta k = sAk - (n+d)k$

- There is no convergence in the long run.
- What’s the growth rate in per capita output?

Graph: The AK Model



Transitional Dynamics: AK Model



Basic Human Capital Model (1)

- In this model, human capital is *deliberately accumulated* (i.e. not the outcome of population growth or exogenously specified technical progress).
- The production is given by:

$$y = k^{\alpha} h^{1-\alpha}$$

where h = human capital (unskilled labor is omitted).

- A fraction *s* of output is saved in order to accumulate capital:

$$\Delta k = sy - d_k k$$

- Another fraction *q* of output is saved to augment the quality of human capital:

$$\Delta h = qy - d_h h$$

Basic Human Capital Model (2)

- Let *r* denote the ratio of human to physical capital in the long run: $r = h/k$
- The growth rate of physical capital can be written as:

$$\Delta k/k = sr^{1-\alpha} - d_k$$

- The growth rate of human capital can be written as:

$$\Delta h/h = qr^{-\alpha} - d_h$$

- Assume $d_k = d_h = 0$. Since *the two growth rates of h and k are the same in the long run*, we have:

$$r = q/s$$

➢ The larger the ratio of saving in *h* relative to that of *k*, the larger is the long-run ratio of *h* to *k*.

Basic Human Capital Model (3)

- In the long run, all variables must growth at the same rate.
- The growth rate of physical capital is:

$$\Delta k/k = sr^{1-\alpha} = s^{\alpha} q^{1-\alpha}$$

- Implications of the model:
  - It is possible to have diminishing returns to k, and yet there is no converge in output per capita.
  - The rate of savings and the rate of investment in human capital affect the growth rate. That is, the growth rate is determined within the model.
  - Here, physical and human capital *together* exhibits constant returns. However, if a third factor (e.g. unskilled labor) is introduced, physical and human capital together would exhibit diminishing returns, and results should be similar to Solow model.

The Lucas (1988) Model

- Lucas (JME, 1988) defines human capital as the skill embodied in workers.
- The production function is given by:

$$Y = AK^{\alpha}(uhN)^{1-\alpha} = AK^{\alpha}(L)^{1-\alpha}$$

where N = total workers

h = stock of human capital (of each worker)

u = proportion of total labor spent on producing output

L = uhN = effective worker

- Human capital grows at a constant rate:

$$\Delta h/h = a(1-u)$$

Where *a* reflects quality or efficiency of education

Lucas Model (cont'd)

- Physical capital accumulation is defined as usual: *ΔK = sY + d.*
- Rewrite the production function in terms of per effective worker:

$$y_e = Ak_e^{\alpha} \qquad (A > 0)$$

- The growth rate of physical capital per effective worker is:

$$\frac{\Delta k_e}{k_e} = \frac{\Delta K}{K} - \frac{\Delta u}{u} - \frac{\Delta h}{h} - \frac{\Delta N}{N}$$

$$\frac{\Delta k_e}{k_e} = sAk_e^{\alpha-1} - d - a(1-u) - n$$

$$\rightarrow \qquad \Delta k_e = sy_e - [d + a(1-u) + n]k_e$$

- At the equilibrium in the long run,

➢ Capital per effective worker is: $k_e^* = [s/(d + a(1-u) + n)]^{1/(1-\alpha)}$

➢ Output per effective worker is: $y_e^* = A[s/(d + a(1-u) + n)]^{\alpha/(1-\alpha)}$

- Note: This simplified version of Lucas model is similar to the Solow Model with augmented labor technology.

Lucas Model (cont'd)

- In the long run, there is no change in physical capital per effective worker. Thus, the growth rate of physical capital per worker can be written as:

$$\frac{\Delta k_e}{k_e} = 0 \rightarrow 0 = \frac{\Delta K}{K} - \frac{\Delta N}{N} - \frac{\Delta h}{h}$$

$$\frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta N}{N} = \frac{\Delta h}{h} = a(1-u)$$

$$\rightarrow \frac{\Delta k}{k} = a(1-u)$$

- Main driver for growth is the growth rate of human capital.
  - Policies that promote knowledge accumulate will result in higher growth rate of output.
- 1-u can be thought of as the production of knowledge.

Growth Model with Externalities (1)

- Suppose there are several firm within the economy.
- Let the production function of each firm be:

$$Y = EK^{\alpha}P^{1-\alpha}$$

where Y = output, K = capital, P = labor employed, E = a measure of overall *external productivity*.

- Now, E is a positive externality generated by the joint capital accumulation of all firms in the economy:

$$E = a(K^*)^{\beta}$$

where a > 0 and β > 0.

- Substitute E in the production function, and we get:

$$Y = a(K^*)^{\beta}K^{\alpha}P^{1-\alpha}$$

Growth Model with Externalities (2)

- Suppose that all the firms are owned by a *benevolent planner* (who values the capital investment of one firm on the overall productivity of other firms).
  - *Positive externalities* created by capital accumulation give a *macroeconomic* production function that may exhibit *increasing returns*.
- Assume all the firms are identical. The "social" production function is:

$$Y = aK^{\alpha+\beta}P^{1-\alpha}$$

- Per capita economic growth is not only positive, also tends to accelerate over time.

Romer's Model (Knowledge Externalities)

- Romer (1990) and Jones (1995) use a model of firms that invest in R&D.
- A firm's R&D raises its profits, but also create a positive externality on other firm (lead to increasing returns overall).
- The production function is given by:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}$$

where A = stock of ideas/knowledge

L_Y = labor used in the production sector

L = total labor: $L = L_Y + L_A$ (L_A = labor used in R&D)

- Capital accumulation equation is given by:

$$\Delta K/K = s_K Y - dK$$

Romer's Model (cont'd)

- Assume the production function for ideas:

$$\Delta A = \delta A^{\phi}(L_A)^{\lambda}, \qquad \delta = \text{a constant}$$

where ϕ = knowledge spillover (ϕ = 0: no new ideas),

λ = originality of idea (i.e. λ = 1: no duplication)

- Suppose the population growth is n, and that $L_A/L = s_R$.
  - $L_Y/L = 1 - s_R$
- Question: What determine the growth rate of A (i.e. growth rate of ideas)?
  - It depends on the parameters ϕ, λ and growth of L_A.
- Define the *growth rate of ideas* as:
  - $g_A = \Delta A/A = \lambda n/(1-\phi)$

Romer's Model (cont'd)

- Case1: assumes $0 < \lambda < 1$ and $\phi < 1$ (Jones,1995).
  - $\Delta A = \delta A^{\phi}(L_A)^{\lambda}$
  - $g_A = \Delta A/A = \lambda n/(1-\phi)$

- Case 2: assumes: $\lambda = 1$ and $\phi = 0$. (Romer, 1990).
  - $\Delta A = \delta L_A$
  - $g_A = \Delta A/A = n$