# Group Homework 2

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 15th 2022 (before midnight /B.E. moodle).

**Note:** Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

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1. Consider the ice cream market in Bangkok. In April, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units, T is the level of temperature in degree Celsius, and P is the price in dollars per unit of ice cream:

$$\text{Demand: } Q = 10000 + 400 \times (T - 30) - 10P$$
$$\text{Supply: } Q = 2000 + 20P$$

Suppose that T = 40 Celsius, find the equilibrium price and quantity of ice cream in April using the Inverse matrix method.

2. Consider a modified version of the IS-LM model where government spending is tied to the level of GDP ($Y$).

$C = C_0 + C_1 Y_d - C_2 r, \qquad 0 < C_1 < 1$

$I = I_0 + I_1 Y - I_2 r, \qquad 0 < I_1 < 1$

$G = G_0 - G_1 Y, \qquad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$L^d = L_0 + L_1 Y - L_2 r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, $M^s$ is the level of money supply, $L^d$ is the level of real money demand, and $r$ is the level of nominal interest rate. Suppose that price is fixed equal to 1. All the coefficients are *non-negative*. Additionally, we assume that $I_1 + C_1 - G_1 < 1$.

a. Discuss about the nature of government behavior. Does the assumption over the behavior of government make sense in practice?
b. What does $C_2$ represent? What does it imply about the behavior of private consumption? Does the assumption make sense?
c. Derive the IS equation. Interpret the meaning of the IS equation. Discuss about the key relation derived from IS equation.
d. Calculate the slope of IS curve. When is the IS curve flat? What does the flat IS curve imply about the sensitivity of real GDP to the interest rate?
e. Use the IS equation and calculate the tax multiplier. How does the tax multiplier depend on slope of IS curve?

f. Derive the LM equation. Interpret the meaning of the LM equation. Discuss about the key relation derived from LM equation.

g. Calculate the slope of LM curve. When is the LM curve flat? What does the flat LM curve imply about the sensitivity of interest rate to the real GDP?

h. Write both IS and LM equations in terms of the matrix representation.

i. Solve for the equilibrium GDP and interest rate $(Y^*, r^*)$ using the Cramer's rule. (*Caution*: you will get <u>**ZERO**</u> if you don't use the Cramer's rule.)

j. Calculate the multiplier of $G_0$ and the multiplier of $M_0$ *on* both $Y^*$ and $r^*$, respectively. Discuss whether the multiplier is bigger or smaller than the case that government spending is purely exogenous, i.e. $G_1 = 0$.

The government is seeking for some advices on fiscal and monetary policy implementation. The goal of the government is to (i) *increase the real GDP (Y) by $100*, while (ii) *keeping the current level of interest rate stayed the same*. (That is, the government was thinking that the country is running into an unemployment situation, but the level of interest rate is now optimal.) Following the storyline given here and all your work that you have done before, answer the next two questions.

k. Can the government successfully achieve both goals by simply relying on *a single type of policy implemented*? That is, to achieve the two goals, would it work to either change the *government expenditure* or *money supply*, but not both at the same time? If yes, *under which conditions*?

l. If the condition that you assumed in (k) does not hold, what would you recommend to the government so that both goals can be simultaneously achieved? (Hint: think about an appropriate mixture of the two policies.)

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$$\text{Demand: } Q = 10000 + 400 \times (T - 30) - 10P$$
$$\text{Supply: } Q = 2000 + 20P$$

Suppose that T = 40 Celsius, find the equilibrium price and quantity of ice cream in April using the Inverse matrix method.

$$Q_D = 10000 + 400(T - 30) - 10P$$

$$T = 40 \rightarrow Q_D = 10000 + 400(40 - 30) - 10P$$

$$Q_D = 14000 - 10P \rightarrow Q + 10P = 14000 \quad \text{— (1)}$$

$$Q_S = 2000 + 20P \rightarrow Q - 20P = 2000 \quad \text{— (2)}$$

$$\begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix} \begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 14000 \\ 2000 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \quad \Big\} \rightarrow \text{To find } X = \frac{B}{A} = A^{-1} \cdot B$$

$$\boxed{A^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

$$A = \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix} \\ \begin{array}{cc} c & d \end{array} \end{array}$$

$$A^{-1} = \frac{1}{-20 - 10} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix} = \frac{1}{-30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{30} & -\frac{1}{30} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{30} & -\frac{1}{30} \end{bmatrix} \begin{bmatrix} 14000 \\ 2000 \end{bmatrix} = \begin{bmatrix} \left(\frac{2}{3} \times 14000\right) + \left(\frac{1}{3} \times 2000\right) \\ \left(\frac{1}{30} \times 14000\right) - \left(\frac{1}{30} \times 2000\right) \end{bmatrix}$$

$$X = \begin{bmatrix} 10000 \\ 400 \end{bmatrix}$$

$\therefore$ $Q_e$ = 10,000 Ice-cream, $P_e$ = 400 dollars #

2. Consider a modified version of the IS-LM model where government spending is tied to the level of GDP ($Y$).

$C = C_0 + C_1 Y_d - C_2 r, \quad 0 < C_1 < 1$

$I = I_0 + I_1 Y - I_2 r, \quad 0 < I_1 < 1$

$G = G_0 - G_1 Y, \quad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$L^d = L_0 + L_1 Y - L_2 r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, $M^s$ is the level of money supply, $L^d$ is the level of real money demand, and $r$ is the level of nominal interest rate. Suppose that price is fixed equal to 1. All the coefficients are *non-negative*. Additionally, we assume that $I_1 + C_1 - G_1 < 1$.

a. Discuss about the nature of government behavior. Does the assumption over the behavior of government make sense in practice?

It seems make sense because when y increase, the goverment doesn't need to spend. However, in reality, the government spending depends on the plan.

b. What does $C_2$ represent? What does it imply about the behavior of private consumption? Does the assumption make sense?

$C = C_0 + C_1 Y_d - C_2 r$

$C_2$ shows sensitivity between r and $C_2$ when r increase, cost of borrowing decrease people borrow less so consumption decrease.

Sign is ⊖ $r \uparrow \rightarrow C \downarrow$, $r \downarrow \rightarrow C \uparrow$ make sense

① $r \uparrow \rightarrow$ saving $\uparrow \rightarrow C \downarrow$

② $r \uparrow \rightarrow$ borrow $\downarrow \rightarrow C \downarrow$

c. Derive the IS equation. Interpret the meaning of the IS equation. Discuss about the key relation derived from IS equation.

$$C = C_0 + C_1 Y_d - C_2 r, \qquad 0 < C_1 < 1$$

$$I = I_0 + I_1 Y - I_2 r, \qquad 0 < I_1 < 1$$

$$G = G_0 - G_1 Y, \qquad 0 < G_1 < 1$$

$$T = T_0$$

IS : $y = C + I + G$

$y = [C_0 + C_1[y - T] - C_2 r] + [I_0 + I_1 y - I_2 r] + G_0 - G_1 y$

$y = C_0 + C_1 y - C_1 T - C_2 r + I_0 + I_1 y - I_2 r + G_0 - G_1 y$

$y - C_1 y - I_1 y + G_1 y = C_0 - C_1 T + I_0 + G_0 - [C_2 + I_2] r$

$(1 - C_1 - T_1 + G_1) y = C_0 - C_1 T + I_0 + G_0 - [C_2 + I_2] r$

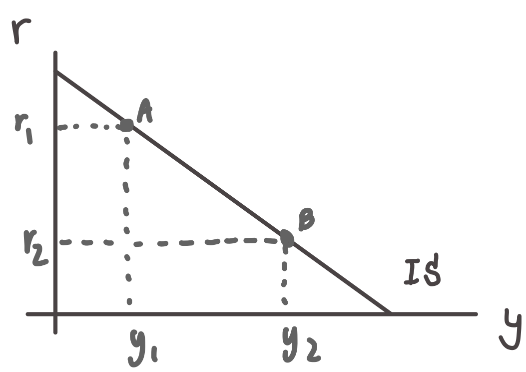
$\rightarrow \quad y = \dfrac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 + G_1} - \left(\dfrac{C_2 + I_2}{1 - C_1 - I + G_1}\right) r$

IS equation : $y = \left[\dfrac{C_0 + I_0 - G_0 - T_0}{1 - I_1 + G_1 - C_1}\right] - \dfrac{(I_2 + C_2)}{[1 - I_1 + G_1 - C_1]} \cdot r$

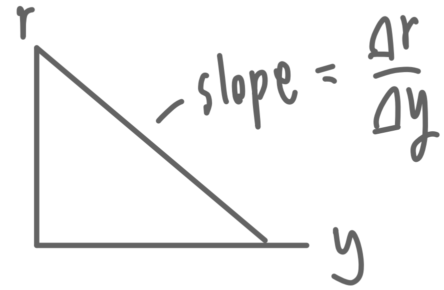
IS : Product Market from equation

$r \uparrow \rightarrow y \downarrow$

$r \uparrow \rightarrow C \downarrow \; I \downarrow \rightarrow y \downarrow$



d. Calculate the slope of IS curve. When is the IS curve flat? What does the flat IS curve imply about the sensitivity of real GDP to the interest rate?




$$y = \left[\frac{C_0 + I_0 - G_0 - T_0}{1 - I_1 + G_1 - C_1}\right] - \frac{(I_2 + C_2)}{[1 - I_1 + G_1 - C_1]} \cdot r$$

$$\text{Slope IS} = \frac{\Delta r}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta r}} = \frac{1}{dy/dr}$$

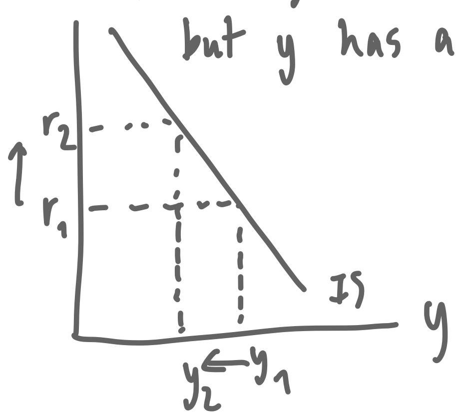
$$\frac{dy}{dr} = -\left[\frac{C_2 + I_2}{1 - I_1 + G_1 - G_1}\right]$$

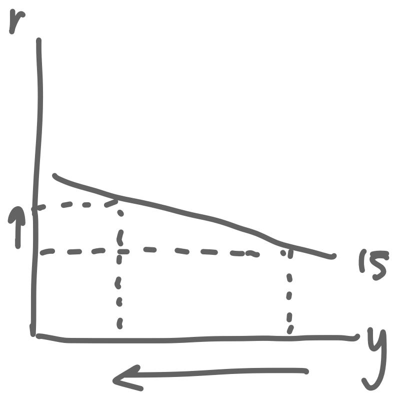
$$\text{Slope (absolute turn)} = \frac{C_2 + I_2}{1 - I_1 - G_1 - C_1}$$

Flat = Slope ↓ when $C_2$, $I_2$ ↓

$[1 - I_1 - G_1 - C_1]$ ↑

r changea a lot but y has a few change




"Less sensitivity"

"more sensitivity"
↓
flat

e. Use the IS equation and calculate the tax multiplier. How does the tax multiplier depend on slope of IS curve?

$$y = \left[\frac{C_0 + I_0 - G_0 - T_0}{1 - I_1 + G_1 - C_1}\right] - \frac{(I_2 + C_2)}{(1 - I_1 + G_1 - C_1)} \cdot r$$

$$\text{Tax multiplier} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{1}{1 - C_1 - I_1 + G_1}$$

Tax Multiplier has same denominator as slope so it depends on slope

f. Derive the LM equation. Interpret the meaning of the LM equation. Discuss about the key relation derived from LM equation.

money S, money D

$$LM: \frac{M^s}{P} = L^d$$

$$\frac{M^o}{P} = L_0 + L_1 y - L_2 r$$

$$y = \frac{MPL_0 + PL_1}{PL_1}$$

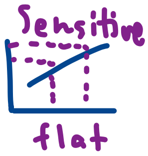
$$y = \frac{M}{PL_1} - \frac{PL_0}{PL_1} + \frac{PL_2 r}{PL_1}$$

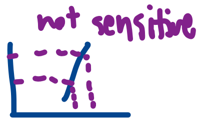
$$\boxed{y = \left[\frac{M}{PL_1} - \frac{L_0}{L_1}\right] + \frac{L_2}{L_1} \cdot r}$$ LM

$r \uparrow \rightarrow y \uparrow$

logic $y \uparrow \rightarrow M^d \uparrow \rightarrow r \uparrow$ $\boxed{y \uparrow r \uparrow}$

g. Calculate the slope of LM curve. When is the LM curve flat? What does the flat LM curve imply about the sensitivity of interest rate to the real GDP?




$$\text{Slope } LM = \frac{dr}{dy}$$

$$\frac{dy}{dr} = \frac{L_2}{L_1}$$

$$\text{So slope } LM = \frac{dr}{dy} = \frac{1}{\left[ L_2 / L_1 \right]} = \frac{L_1}{L_2}$$

LM will flat if $L_1$ decrease or $L_2$ increase

h. Write both IS and LM equations in terms of the matrix representation.

$$IS : y = \frac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 - G_1} - \frac{C_2 + I_2}{1 - C_1 - I_1 - G_1} \cdot r$$

$$y + \frac{C_2 + I_2}{1 - C_1 - I_1 - G_1} \cdot r = \frac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 - G_1} \quad - ①$$

$$LM : y = \frac{M_0}{L_1 P} - \frac{L_0}{L_1} + \frac{L_2}{L_1} r$$

$$y - \frac{L_2}{L_1} r = \frac{M_0}{L_1 P} - \frac{L_0}{L_1} \quad - ②$$

$$\rightarrow \begin{bmatrix} 1 & \frac{C_2 + I_2}{1 - C_1 - I_1 - G_1} \\ 1 & -\frac{L_2}{L_1} \end{bmatrix} = \begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} \frac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 - G_1} \\ \frac{M_0}{L_1 P} - \frac{L_0}{L_1} \end{bmatrix}$$

i. Solve for the equilibrium GDP and interest rate $(Y^*, r^*)$ using the Cramer's rule. (*Caution*: you will get **ZERO** if you don't use the Cramer's rule.)

assume $= A$

C

$$\begin{bmatrix} 1 & \dfrac{C_2 + I_2}{1 - C_1 - I_1 - G} \\ 1 & -\dfrac{L_2}{L_1} \end{bmatrix} \begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} \dfrac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 - G_1} \\ \dfrac{M_0}{L_1 P} - \dfrac{L_0}{L_1} \end{bmatrix}$$

B D

$$\begin{bmatrix} 1 & A \\ 1 & B \end{bmatrix} \begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix}$$

put ABCD

$$y = \frac{\begin{vmatrix} C & A \\ D & B \end{vmatrix}}{\begin{vmatrix} 1 & A \\ 1 & B \end{vmatrix}} = \frac{CB - DA}{B - A}$$

$$r = \frac{\begin{vmatrix} 1 & C \\ 1 & D \end{vmatrix}}{\begin{vmatrix} 1 & A \\ 1 & B \end{vmatrix}} = \frac{D - C}{B - A}$$

$$y = \frac{\left[\dfrac{C_0 - C_1 T + I_0 + G_0}{1 - C_1 - I_1 - G_1}\right]\left[-\dfrac{L_2}{L_1}\right] - \left[\dfrac{M_0}{L_1 P} - \dfrac{L_0}{L_1}\right]\left[\dfrac{C_2 + I_2}{1 - C_1 - I_1 - G_1}\right]}{\left[-L_2/L_1\right] - \left[\dfrac{C_2 + I_2}{1 - C_1 - I_1 - G_1}\right]}$$

$$r = \frac{\left[\dfrac{M_0}{L_1 P} - \dfrac{L_0}{L_1}\right] - \left[\dfrac{C_0 - CT + I_0 + G_0}{1 - C_1 - I_1 - G_1}\right]}{\left[-L_2/L_1\right] - \left[\dfrac{C_2 + I_2}{1 - C_1 - I_1 - G_1}\right]}$$

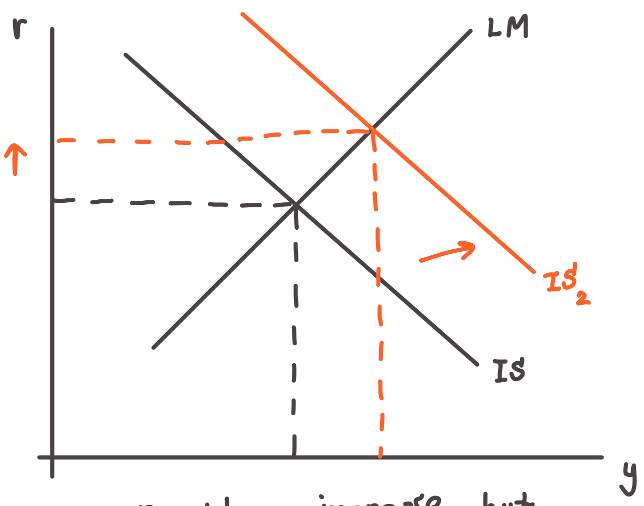
j. Calculate the multiplier of $G_0$ and the multiplier of $M_0$ *on* both $Y^*$ and $r^*$, respectively. Discuss whether the multiplier is bigger or smaller than the case that government spending is purely exogenous, i.e. $G_1 = 0$.

The government is seeking for some advices on fiscal and monetary policy implementation. The goal of the government is to (i) *increase the real GDP (Y) by $100*, while (ii) *keeping the current level of interest rate stayed the same*. (That is, the government was thinking that the country is running into an unemployment situation, but the level of interest rate is now optimal.) Following the storyline given here and all your work that you have done before, answer the next two questions.

k. Can the government successfully achieve both goals by simply relying on *a single type of policy implemented*? That is, to achieve the two goals, would it work to either change the *government expenditure* or *money supply*, but not both at the same time? If yes, *under which conditions*?

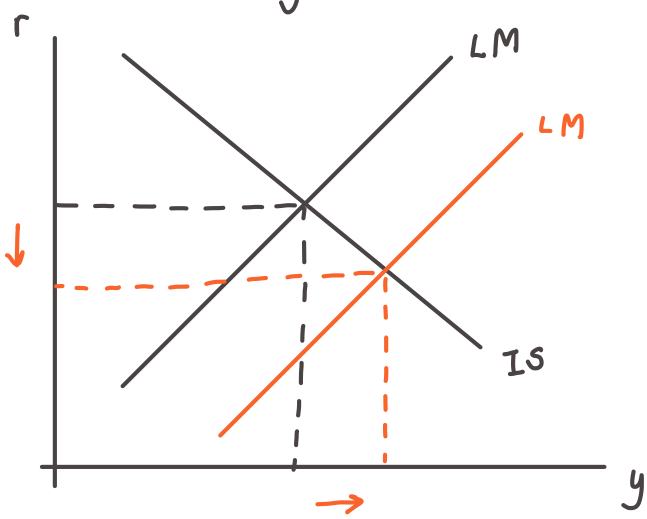
No

If use only fiscal (G ↑)



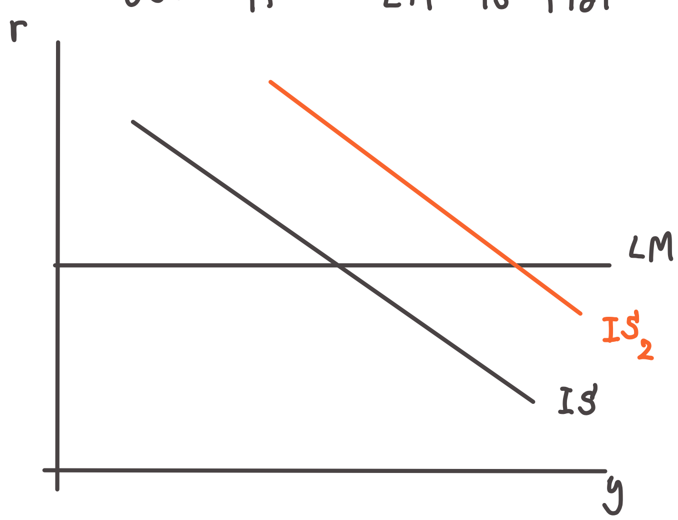

→ result y increase but r increase

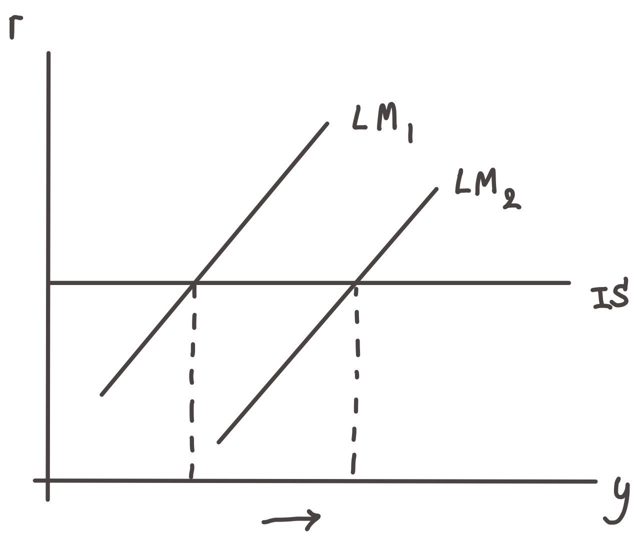
use only Monetary (M↑)




result y increase but r decrease

Yes if LM is Flat



l. If the condition that you assumed in (k) does not hold, what would you recommend to the government so that both goals can be simultaneously achieved? (Hint: think about an appropriate mixture of the two policies.)

the Government must increase Government spending

the central Bank must increase Money Supply