

Present Value



Which one you would prefer 100 baht today or 100 baht one year from now?
Why?

Present Value of a Single Sum

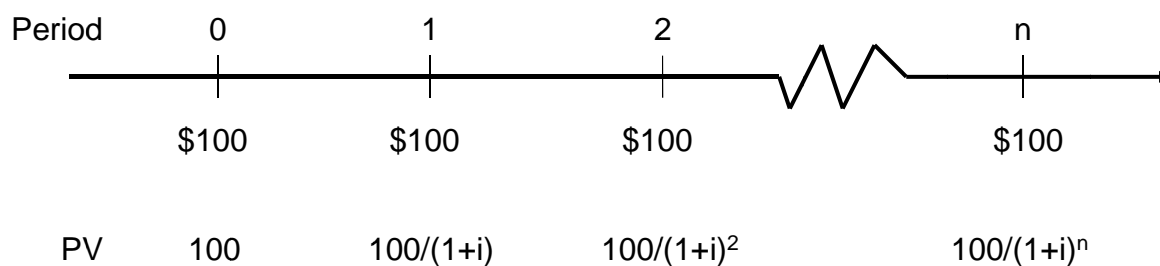
$$PV_0 = \frac{FV_n}{(1+i)^n}$$

where PV_0 = present value at time 0
 FV_n = future value at time n
 i = interest rate per period
 n = numbers of period

A dollar paid to you one year from now is less valuable than a dollar paid to you today

- A dollar deposited today can earn interest and become $\$1 \times (1+i)$ one year from today.

Cannot directly compare payments scheduled in different points in the time line



Other names for interest rate

- Discount rate
- Opportunity cost
- Rate of return
- Required rate of return
- Expected rate of return
- Cost of capital

Simple Annual Rate versus Effective Annual Rate

$$EAR = \left[1 + \frac{SAR}{m} \right]^m - 1$$

where EAR = effective annual rate or effective annual yield
 SAR = simple annual rate or annual percentage rate
 m = numbers of frequency of interest payment in a year

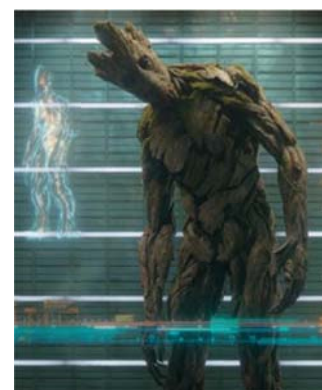
Exercise 1

Groot would like to deposit 100 baht with the bank for 1 year, find the total amount that Groot will have under each of the following interest rate:

- 10% interest compounded annually
- 10% interest compounded semi-annually
- 10% interest compounded quarterly
- 10% interest compounded monthly

Also, disaggregate the principal and interest amounts for Groot.

What is interest on interest?



Exercise 2

Seiko deposited 780,000 baht in a savings account that pays 6.88 percent interest, compounded quarterly, planning to use it to ease her life during her retirement. Eighteen months later, she met with Takeshi whom she would like to spend the rest of her life with. So, rather than continue to plan for her retirement alone, she starts to build her family and closes out her account. However, Takeshi needed Seiko to move to Japan with him. How much money will Seiko receive in yen, given 0.3035 baht/yen?



Exercise 3

After Phantom had kidnapped Christine, he sent Raoul a message saying that if Raoul wanted Christine back, he just gave Phantom 500,000 French francs (a monetary unit used during that period) in return. However, Raoul had cash only 30% of the amount Phantom asked. Thus, he borrowed the rest from his business, the Opera House. He is required to repay a lump sum of 532,306.25 francs three years from borrowing. What was the nominal interest rate that the Opera House charged Raoul¹ ?



Chatuporn Tangkathach

¹ written by Obrom Chaowalerd adapted by Chatuporn Tangkathach

Exercise 4

Bart is computing the present value (today) and future value (at the end of year 4) of the following stream of cash flows. Given interest rate of 12.75 percent (1) compounded annually and (2) compounded semi-annually, what are Bart's answers?

<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>
1,500 baht	1,700 baht	2,300 baht



Exercise 5

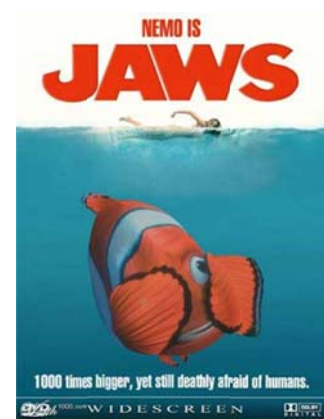
Tony Stark deposits 2 accounts separately today each USD100,000. For first account, he expects to receive 2.75 percent return compounded quarterly. Another account will give him 3.50 percent compounded every six-month. After he deposited these 2 accounts for 7 years, he will combine them into single account and invest for the next 2 years. What will be the rate of return per year compounded monthly for these 2 years, if at the end of 2 years, Tony receives total of USD270,000?



Exercise 6

Nemo prepared for FN 211 exam, he already computed the total future value (at $t = 5$) of stream of cash flow which he got £23,467.41 when compounded at 12.50 percent annually. Then, he forgot second year cash flows. Help Nemo to find the value of the missing ($t = 2$) cash flow.

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
0	£2,500	£ ?	£7,000	£7,000	£7,000



Exercise 7

From prior exercise, what must be the interest rate given cash flows in year 2 equals zero and sum of present values of these cash flows is £15,000?

Future Value of Annuity

$$FV_n = CF \left[\frac{(1+i)^n - 1}{i} \right]$$

Present Value of Annuity

$$PV_0 = CF \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

where PV_0 = present value at time 0
 FV_n = future value at time n
CF = cash flow per period in the future
 i = interest rate per period
 n = numbers of period

Exercise 8

Piglet intends to collect money for retirement. If Piglet deposits £190 at the end of each of the next 50 years into an account paying 9.00 percent interest, annually compounded, how much money will Piglet have in the account in 50 years?



Exercise 9

Panda wants to have 120,000 pound in his savings account five years from now; he is prepared to make equal deposits into the account at the end of each year. If the account pays 12.75 percent interest, annually compounded, what amount must he deposit each year?



Exercise 10

Beginning three months from now, Tintin wants to be able to withdraw 10,000 dollars each quarter from his bank account to cover his personal expenses over the next five years. If the account pays 16.00 percent interest, compounded quarterly, how much Tintin need to have in his bank account today to meet his expense needs over the next five years?



Exercise 11

Peter would like to deposit 550 baht at the end of each year for 3 years. What are the total present value and future value, given 6.25 percent interest rate, quarterly compounding? Compute effective annual rate and use this EAR to recompute future value? Explain your answers.



Exercise 12

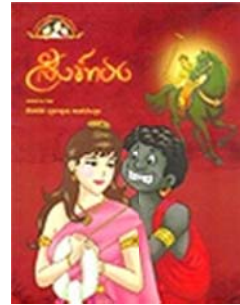
What is the future value of 1,000 baht per year for 20 years, given interest of 5.25 percent per year, semi-annually compounded ?

Exercise 13

What is the future value of 1,000 baht every six months for 40 years, given interest of 3.50 percent per year, quarterly compounded ?

Exercise 14

Sungthong plans to marry Rojana in the near future. He saves 40% of his annual income in a time-deposit account, which earns 12.40 percent interest, compounded quarterly. The first and last deposits are made 1 year from today and at his marriage, respectively. His annual income is constant at 100,000 baht. If the dowry to be paid on the wedding day is 1,615,206.06 baht, when will Sungthong marry Rojana²?



Exercise 15

When Luke was born, Darth Vader, his father, gave him 12,000 baht which he deposited for Luke since then. Up to the time Luke was 15 years old, Luke started to work and adding his savings in the same account that his father opened for him. Luke deposited 750 baht per year every year. What will be the total amount in Luke's bank account at his age of 25 years old? Given the interest rate in Luke's bank account is 9.50 percent per year, annually compounded.



² written by Obrom Chaowalerd adapted by Chatuporn Tangkathach

Exercise 16

Nichkhun would like to retire at the age of 50 years old, after which he expects to live until he is 80 years old. During his retirement, Nichkhun expects to receive 330,000 baht per year. Given his purpose, today, Nichkhun is 18 years old. How much equal amount of money Nichkhun needed to deposit each year from now until he is 50 years old in order to finalise his objective? Nichkhun will deposit in the bank account which earns 7.00 percent per year.



Perpetuity

$$PV_0 = \frac{CF}{i}$$

where PV^0 = present value at time 0

CF = cash flow per period

i = interest rate per period

Exercise 17

Kwun is considering the purchase of 40 square yards of land. Ream, his girlfriend, indicates that if the land is used for cattle grazing, this will produce cash flow of 25,000 baht per year per square yard indefinitely. If Kwun requires a return of 25 percent on this investment, what is the most that Kwun should be willing to pay for the land?



Exercise 18

Aladdin would like to set aside an amount of cash in the specific account that aims to provide an even stream of donation to various foundations. If the account pays 5 percent interest, annually compounded, and Aladdin would like to donate 500 baht per year forever, starting the first donation today, what is the appropriate amount he needs to deposit today? Also, prove that this amount satisfied his donation desire.



Growing Perpetuity

$$PV_0 = \frac{CF_1}{i - g}$$

where PV^0 = present value at time 0

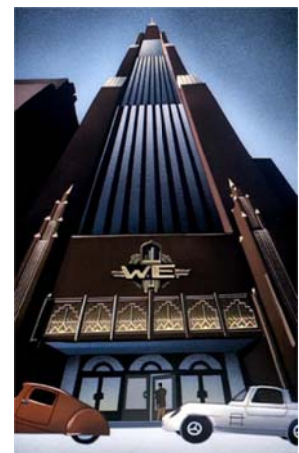
CF^1 = cash flow in period 1

i = interest rate per period

g = growth rate per period

Exercise 19

Wayne Enterprise is considering investing on an advanced technology. This technology will be able to generate income in the near term. Bruce Wayne, the owner, anticipates his first annual cash flow from the technology to be 520,000 baht received two years from today. Subsequent annual cash flows will grow at 5.25 percent in perpetuity. What is the maximum investment Wayne Enterprise should invest in this technology? Given required return on this investment equals to 11.80 percent.



Exercise 20

Krongkaew won Sri-Siam contest. She would be award the prize; however, 5 sponsors offered her different prize. She was able to accept one prize only. Which prize, from the following sponsors, she should choose?



I will give her 30,000 baht today and another amount 80,000 baht at the end of 8 years from now.

Tharathorn

I will give her 11,000 baht per year for 9 years, and the first amount will start right now.



Bhawornruth



I will give her, 17,000 baht every six-month for the period of 12 years, but the first 17,000 baht will start 10 years from today.

Buddhiphat

I will give her, 7,500 baht per year only, but I will give her same amount 7,500 baht per year forever. The first 7,500 baht, I will give to her immediately.



Radchanond



I will give her, six-month from now the first 3,000 baht and every six-month I will give her more where the amount will grow at the 6.00 percent per year. I will give her for 20 years counting from today.

Ronnaphee

Given Krongkaew has 12.00 percent per annum, opportunity cost, semi-annually compounded.

Exercise

21. You are saving for the college education of your two children. They are two years apart in age; one begins college 15 years from today and the other will begin 17 years from today. You estimate your children's college expenses to be 230,000 baht per year per child, payable at the beginning of each school year. The annual interest rate is 5.50 percent. How much money must you deposit in an account each year to fund your children's education? You deposit begin one year from today. You will make your last deposit when your oldest child enters college. Assume four years of college.
22. You're prepared to make monthly payments of 1,000 baht, beginning at the end of this month, into an account that pays 10.00 percent interest compounded monthly. How many payments will you have made when your account balance reaches 150,000 baht?
23. Your job pays you only once a year for all the work you did over the previous 12 months. Today, December 31st, you just received your salary of 400,000 baht, and you plan to spend all of it. However, you want to start saving for retirement beginning next year. You have decided that one year from today you will begin depositing 2.00 percent of your annual salary in an account that will earn 8.00 percent per year. Your salary will increase at 4.00 percent per year throughout your career. How much money will you have on the date of your retirement 40 years from today?
24. Bilbo Baggins wants to save money to meet three objectives. First, he would like to be able to retire 30 years from now with a retirement income of €25,000 per month for 20 years, with the first payment received 30 years and 1 month from now. Second, he would like to purchase a cabin in Riverdell in 10 years at an estimated cost of €350,000. Third, after he passes on at the end of the 20 years of withdrawals, he would like to leave an inheritance of €750,000 to his nephew Fredo. He can afford to save €2,100 per month for the next 10 years. If he can earn an 11.00 percent EAR before he retires and an 8.00 percent EAR after he retires, how much will he have to save each month in years 11 through 30?
25. You are serving on a jury. A plaintiff is suing the city for injuries sustained after a freak street sweeper accident. In the trial, doctors testified that it will be five years before the plaintiff is able to return to work. The jury has already decided in favour of the plaintiff. You are the jury and propose that the jury give the plaintiff an award to cover the following: (1) the present value of two years back pay. The plaintiff's annual salary for the last two years would have been 400,000 baht and 430,000 baht, respectively. (2) the present value of five years future salary. You assume the salary will be 450,000 per year. (3) 1,000,000 for pain and suffering. (4) 200,000 baht for court costs. Assume that the salary payments are equal amounts paid at the end of each month. If the interest you choose is a 9.00 percent EAR, what is the size of the settlement?
26. You have just won the lottery. You will receive 1,000,000 baht today, and then receive 40 payments of 500,000 baht. These payments will start one year from now and will be paid every six months. A representative from Get Rich Now Corp. has offered to purchase all the payments from you for 10 million baht. If the appropriate interest rate is a 9.00 percent EAR compounded daily, should you take the offer? Assume there are 12 months in a year, each with 30 days.
27. What is the value of an investment that pays AUD6,700 every other year forever, if the first payment occurs one year from today and the discount rate is 13.00 percent compounded daily? What is the value today if first payment occurs four years from today?
28. What is the equation for the present value of a growing perpetuity with a payment of C one period from today if the payments grow by C each period?