

Instructions

- (1) Please read the instruction carefully. Also take this habit with you into the exam room.
- (2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at least a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
- (3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

- (1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
- (2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
- (3) Default decimal point is 4.
- (4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
- (5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
- (6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

- (1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
- (2) Login to BE Moodle, head into the course, then the assignment topic.
- (3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

For all questions, answer up to 4 decimal places

Question 1. (15 points) Given this information

$$\begin{aligned}
 n &= 18 & \sum_{i=1}^n X_i &= 388.00 & \sum_{i=1}^n Y_i &= 50.90 \\
 \sum_{i=1}^n (X_i)^2 &= 9,620.00 & \sum_{i=1}^n X_i Y_i &= 1,254.90 \\
 \sum_{i=1}^n (X_i - \bar{X})^2 &= 211.00 & \sum_{i=1}^n (Y_i - \bar{Y})^2 &= 2.5844 \\
 \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) &= 20.58 & \sum_{i=1}^n \hat{u}_i^2 &= 0.5781
 \end{aligned}$$

Use the above sample information to answer all the following questions. Show explicitly all formulas and calculations.

- From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$, **find the estimators** of β_1 and β_2 with OLS method. Interpret the intercept and slope coefficients.
- Compute the value of R^2 and explain its meaning.
- If $X_i = 30$, estimate the value of $\hat{Y}_i$ and explain its meaning.
- Calculate the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$.
- What are the 90-percent confident intervals for β_2 ? Interpret the meaning.
- Test the hypothesis whether the slope coefficients are different from zero at 0.05 level of significance.

a) OLS $y = b_1 + b_2 x + e$

$$b_2 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} \quad \bar{x} = \frac{\sum x}{N}$$

$$b_1 = \bar{y} - b_2 \bar{x} \quad \bar{y} = \frac{\sum y}{N}$$

Step 1 $\bar{x} = \frac{\sum x}{N} = \frac{388}{18} = 21.5555$

$$\bar{y} = \frac{\sum y}{N} = \frac{50.90}{18} = 2.8278$$

Step 2 in b_2

$$b_2 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{20.58}{211.00} = 0.0975$$

Step 3

$$b_1 = \bar{y} - b_2 \bar{x} = 2.8278 - 0.0975 (21.5555)$$

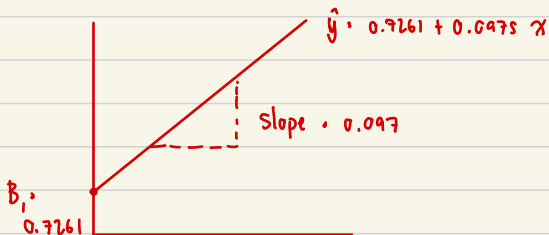
$$b_1 = 0.7261$$

$$\therefore \hat{y} = 0.7261 + 0.0975 x \text{ [Regression]}$$

(slope)

$B_2 = 0.0975$, if x changes by 1 unit, y will change by 0.0975 unit

$B_1 = 0.7261$; if $x = 0$, $\hat{y} = 0.7261$



b)

$$R^2 = \frac{ESS}{TSS}$$

$$R^2 = 1 - \frac{RSS}{TSS}$$

$$ESS = \sum (\hat{y} - \bar{y})^2 \quad TSS = \sum (y - \bar{y})^2 \quad RSS = \sum u^2 = \sum (\hat{y} - y)^2$$

$$R^2 = 1 - \frac{0.5781}{2.5844} = 0.7763$$

* The variation of y can be explained by $x = 77.63\%$

c)

$$\hat{y} = 0.7261 + 0.0973 x$$

$$x = 30$$

$$\hat{y} = 0.7261 + 0.0973 (30)$$

$$\hat{y} = 3.6511$$

d)

$$\text{Var}(\hat{u}) = \frac{\sum \hat{u}^2}{n-2} = \frac{0.5871}{18-2} = 0.0361$$

$$\text{Var}(b_1) = \frac{\text{Var}(\hat{u})}{\sum (x - \bar{x})^2} = \frac{0.0361}{211.00} = 0.00017$$

$$\text{Var}(b_1) = \frac{\text{Var}(\hat{u}) \cdot \sum x^2}{n \cdot \sum (x - \bar{x})^2} = \frac{0.0361 (9620.00)}{18 (211.00)} = 0.0914$$

Confident interval B^2

$$= b \pm \left[t_{\frac{\alpha}{2}, n-2} \right] \cdot \frac{S(b)}{\sqrt{n}}$$

$$\text{Step 1} \quad \alpha = 0.1 / 2 = 0.05$$

$$\text{Step 2} \quad SD(B) = \sqrt{\text{Var}(B_1)}$$

$$SD(b_1) = \sqrt{0.00017}$$

$$SD(b_1) = 0.0130$$

$$\text{Step 3} \quad \text{Con} = 0.0873 \pm 1.746 \cdot \frac{0.0130}{\sqrt{18}}$$

$$= 0.0921, 0.1028$$

$\therefore$ according to the regression, the true B_2 is between 0.0921 and 0.1028 at 90% significant level.

f). Test hypothesis

$$\text{Step 1 : } H_0 : B_2 = 0$$

$$H_1 : B_2 \neq 0$$

$$\text{Step 2 : } \alpha/2 = 0.05/2 = 0.025$$

Step 3 : t-test

$$t = \frac{\hat{b} - B}{\sqrt{\frac{\text{var}(B)}{N}}}$$

$$t = \frac{0.0975 - 0}{\sqrt{\frac{0.3122}{18}}}$$

$$t = 1.3247$$

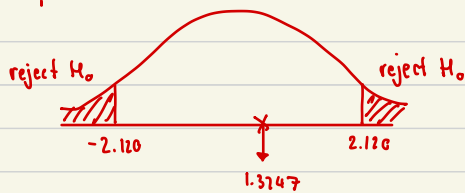
$$\text{Step 4 : } df = n - 2$$

$$= 18 - 2$$

$$= 16$$

$$t_{0.025, 16} = 2.120$$

Step 5



$\therefore$ fail to reject H_0 at 0.05 significant level

Question 2. Using the 2015 Health and Welfare Survey from the National Statistical Office, a simple linear regression is modeled as follows,

$$outp_i = \beta_1 + \beta_2 age_i + u_i$$

where $outp_i$ is how many times person i has visited hospital in 2015, from 0 to 7 times
 age_i is how old is person i , from 0 to 97 years.

We assume that both $outp_i$ and age_i are continuous, the estimation results in the following table. Answer the following questions and show your work.

Source	SS	df	MS	Number of obs	=	27,886
Model	77.5444409	1	77.5444409	F(1, 27884)	=	186.96
Residual	11565.0627	27,884	.414756231	Prob > F	=	0.0000
				R-squared	=	0.0067
				Adj R-squared	=	0.0066
Total	11642.6072	27,885	.417522223	Root MSE	=	.64402

outp	Coefficient	Std. err.	t	P> t	[95% conf. interval]
age	.0031338	.0002292	Omitted		.0026846 .003583
_cons	.4279898	.0140339			.4004828 .4554969

- Test if both parameters are significantly different from zero or not. Use $\alpha = 0.05$.
- Interpret the meaning of $\hat{\beta}_2$. Does the sign of $\hat{\beta}_2$ make economic sense? Explain.
- If $outp_i$ is turned into natural logarithmic scale ($\ln$), how would you reinterpret the relationship between $\hat{\beta}_2$ and $\widehat{outp}_i$, assumed that the given coefficient given in the table above can be used to interpret this new functional form.
- If age_i variable is divided by 10, how does it affect both the coefficients, standard errors, and confidence intervals? Answer the changes of both the constant and slope (if there is).
- Find the confidence interval of mean prediction at the age of 50 years old, given that $var(\hat{Y}_0) = 0.00002$ and $\alpha = 0.01$.

Question 3. Discuss in a short paragraph why the confidence interval for both the mean prediction and individual prediction get larger as the X_0 is further away from $\bar{X}$.

Question 2. Using the 2015 Health and Welfare Survey from the National Statistical Office, a simple linear regression is modeled as follows,

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$$outp_i = \beta_1 + \beta_2 age_i + u_i$$

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STATA

Source	SS	df	MS	Number of obs	=	27,886
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Total	11642.6072	27,885	.417522223	Root MSE	=	.64402

	Coefficient	Std. err.	t	P> t	[95% conf. interval]
age	.0031338	.0002292		.0026846	.003583
_cons	.4279898	.0140339	Omitted	.4004828	.4554969

①

②

Constant

$$outp = B_1 + B_2 age + u$$

$$B_2 = 0.0031338 \text{ หรือ } \frac{\partial outp}{\partial age}$$

$$B_1 = 0.4279898 \text{ หรือ } \frac{\partial outp}{\partial \text{constant}}$$

(Se หรือ มาตรฐานผิดพลาด)

Standard error = มาตรฐานการประมาณค่า, มาตรฐานผิดพลาด

$$\text{Regression: } outp = 0.4279898 + 0.0031338 age$$

$$Se(B_1) = 0.014 \quad Se(B_2) = 0.000229$$

test hypothesis 6 ขั้นตอน

a) Test if both parameters are significantly different from zero or not. Use $\alpha = 0.05$.

Significant level

test b_1

Step 1 set hypothesis

$$H_0: B_1 = 0$$

$$H_1: B_1 \neq 0$$

Step 2: Significant level

Significant level = 0.05

two tails

$$\text{Significant level} / 2 = 0.025$$

Step 3: t-test

$$t\text{-test} = \frac{\hat{B}_1 - B_1}{Se(B_1)}$$

$$t\text{-test} = \frac{0.4279898 - 0}{0.014}$$

$$t\text{-test} = 30.56$$

$$df = n - \text{จำนวน } B$$

$$df = 27886 - 2 = 27884$$

Step 4 T- table

0.025, df = 27884

= 1.96

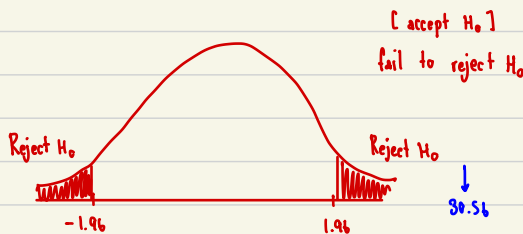
Table 2: T-Distribution Table



$P(t > 2.23) = 0.05$ d.f. = 10

df	0.25	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.559
2	0.818	1.061	1.385	1.886	2.920	5.959	18.858	39.193	77.085	155.990	315.992
3	0.765	0.978	1.250	1.638	2.353	3.182	4.841	6.908	9.925	14.081	21.478
4	0.741	0.941	1.190	1.533	2.132	2.776	4.291	5.774	8.591	12.707	18.447
5	0.727	0.920	1.156	1.476	2.015	2.571	3.982	5.401	8.094	11.578	16.756
6	0.718	0.908	1.134	1.440	1.943	2.447	3.747	5.208	7.879	11.345	16.447
7	0.711	0.898	1.129	1.435	1.925	2.365	3.579	5.091	7.692	11.154	16.224
8	0.706	0.893	1.126	1.431	1.909	2.306	3.476	4.983	7.591	11.034	16.106
9	0.703	0.889	1.123	1.428	1.895	2.262	3.402	4.903	7.501	10.930	16.009
10	0.700	0.887	1.121	1.426	1.882	2.228	3.341	4.838	7.433	10.849	15.925
11	0.697	0.885	1.119	1.424	1.871	2.201	3.291	4.782	7.374	10.776	15.851
12	0.695	0.883	1.118	1.423	1.861	2.179	3.243	4.733	7.321	10.713	15.784
13	0.694	0.882	1.117	1.422	1.852	2.160	3.204	4.689	7.272	10.655	15.722
14	0.693	0.881	1.116	1.421	1.844	2.144	3.172	4.648	7.226	10.601	15.664
15	0.691	0.880	1.115	1.420	1.837	2.130	3.144	4.610	7.183	10.551	15.610
16	0.690	0.879	1.114	1.419	1.831	2.118	3.119	4.575	7.142	10.504	15.560
17	0.689	0.878	1.113	1.418	1.825	2.107	3.096	4.543	7.103	10.460	15.513
18	0.688	0.877	1.112	1.417	1.820	2.096	3.075	4.513	7.066	10.418	15.468
19	0.688	0.876	1.111	1.416	1.815	2.086	3.055	4.484	7.031	10.378	15.425
20	0.687	0.875	1.110	1.415	1.811	2.077	3.036	4.456	7.000	10.340	15.384
21	0.686	0.875	1.109	1.414	1.807	2.069	3.018	4.429	6.969	10.303	15.344
22	0.686	0.874	1.108	1.413	1.803	2.062	3.001	4.403	6.939	10.267	15.305
23	0.685	0.873	1.107	1.412	1.800	2.055	2.985	4.378	6.910	10.231	15.267
24	0.685	0.872	1.106	1.411	1.797	2.049	2.969	4.354	6.882	10.196	15.230
25	0.684	0.871	1.105	1.410	1.794	2.043	2.954	4.330	6.855	10.161	15.194
26	0.684	0.871	1.104	1.409	1.791	2.038	2.939	4.307	6.829	10.127	15.158
27	0.683	0.870	1.103	1.408	1.788	2.033	2.924	4.284	6.803	10.093	15.123
28	0.683	0.869	1.102	1.407	1.785	2.028	2.910	4.261	6.778	10.059	15.088
29	0.682	0.869	1.101	1.406	1.782	2.023	2.895	4.239	6.753	10.025	15.053
30	0.682	0.868	1.100	1.405	1.779	2.019	2.881	4.217	6.729	9.991	15.019
40	0.681	0.867	1.099	1.404	1.776	2.015	2.867	4.195	6.705	9.957	14.985
50	0.679	0.866	1.098	1.403	1.773	2.012	2.854	4.173	6.682	9.923	14.951
60	0.679	0.865	1.097	1.402	1.770	2.009	2.841	4.151	6.659	9.889	14.917
80	0.678	0.864	1.096	1.401	1.767	2.006	2.828	4.129	6.636	9.855	14.883
100	0.677	0.863	1.095	1.400	1.764	2.004	2.815	4.107	6.613	9.821	14.849
1000	0.677	0.863	1.095	1.400	1.764	2.004	2.815	4.107	6.613	9.821	14.849
∞	0.674	0.842	1.091	1.398	1.645	1.960	2.576	3.837	3.990	3.291	3.291

Step 5



Reject H_0 (H_0 is true)

Step 6 Summarize

B_1 is different from Zero at 0.03 significant level *

test b_2

Step 1 : set hypothesis

$$H_0 : B_1 = 0$$

$$H_1 : B_1 \neq 0$$

Step 2

Significant level = 0.05

Step 3 t - test

$$= \frac{\hat{B}_2 - B_2}{Se(B)}$$

$$= \frac{0.0031338 - 0}{0.001792}$$

$$df = n - \text{inuu } B = 27886 - 2 = 27884$$

$$= 13.6728$$

Step 4 : t - table

0.025, 27884

= 1.960

Table 2: T-Distribution Table



$P(t > 2.23) = 0.05$ d.f. = 10

df	0.25	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.559
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7	0.711	0.898	1.129	1.435	1.925	2.365	3.579	5.091	7.692	11.154	16.224
8	0.706	0.893	1.126	1.431	1.909	2.306	3.476	4.983	7.591	11.034	16.106
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10	0.700	0.887	1.121	1.426	1.882	2.228	3.341	4.838	7.433	10.849	15.925
11	0.697	0.885	1.119	1.424	1.871	2.201	3.291	4.782	7.374	10.776	15.851
12	0.695	0.883	1.118	1.423	1.861	2.179	3.243	4.733	7.321	10.713	15.784
13	0.694	0.882	1.117	1.422	1.852	2.160	3.204	4.689	7.272	10.655	15.722
14	0.693	0.881	1.116	1.421	1.844	2.144	3.172	4.648	7.226	10.601	15.664
15	0.691	0.880	1.115	1.420	1.837	2.130	3.144	4.610	7.183	10.551	15.610
16	0.690	0.879	1.114	1.419	1.831	2.118	3.119	4.575	7.142	10.504	15.560
17	0.689	0.878	1.113	1.418	1.825	2.107	3.096	4.543	7.103	10.460	15.513
18	0.688	0.877	1.112	1.417	1.820	2.096	3.075	4.513	7.066	10.418	15.468
19	0.688	0.876	1.111	1.416	1.815	2.086	3.055	4.484	7.031	10.378	15.425
20	0.687	0.875	1.110	1.415	1.811	2.077	3.036	4.456	7.000	10.340	15.384
21	0.686	0.875	1.109	1.414	1.807	2.069	3.018	4.429	6.969	10.303	15.344
22	0.686	0.874	1.108	1.413	1.803	2.062	3.001	4.403	6.939	10.267	15.305
23	0.685	0.873	1.107	1.412	1.800	2.055	2.985	4.378	6.910	10.231	15.267
24	0.685	0.872	1.106	1.411	1.797	2.049	2.969	4.354	6.882	10.196	15.230
25	0.684	0.871	1.105	1.410	1.794	2.043	2.954	4.330	6.855	10.161	15.194
26	0.684	0.871	1.104	1.409	1.791	2.038	2.939	4.307	6.829	10.127	15.158
27	0.683	0.870	1.103	1.408	1.788	2.033	2.924	4.284	6.803	10.093	15.123
28	0.683	0.869	1.102	1.407	1.785	2.028	2.910	4.261	6.778	10.059	15.088
29	0.682	0.869	1.101	1.406	1.782	2.023	2.895	4.239	6.753	10.025	15.053
30	0.682	0.868	1.100	1.405	1.779	2.019	2.881	4.217	6.729	9.991	15.019
40	0.681	0.867	1.099	1.404	1.776	2.015	2.867	4.195	6.705	9.957	14.985
50	0.679	0.866	1.098	1.403	1.773	2.012	2.854	4.173	6.682	9.923	14.951
60	0.679	0.865	1.097	1.402	1.770	2.009	2.841	4.151	6.659	9.889	14.917
80	0.678	0.864	1.096	1.401	1.767	2.006	2.828	4.129	6.636	9.855	14.883
100	0.677	0.863	1.095	1.400	1.764	2.004	2.815	4.107	6.613	9.821	14.849
1000	0.677	0.863	1.095	1.400	1.764	2.004	2.815	4.107	6.613	9.821	14.849
∞	0.674	0.842	1.091	1.398	1.645	1.960	2.576	3.837	3.990	3.291	3.291

Step 5



Step 6

B_2 is different from zero at 0.05 significant level

b) Interpret the meaning of β_2 . Does the sign of β_2 make economic sense? Explain.

Theory $\ln y$ $\log$ $\ln x$

①. linear $y = B_1 + B_2 x + u$

if x change by 1 unit y will change B_2 unit

② Double log: $\ln y = B_1 + B_2 \ln x + u$

if x change by 1%, y will change B_2 %

③ Linear log: $y = B_1 + B_2 \ln x$

if x change by 1%, y will change $B_2 / 100$ unit

④ log - lin $\ln y = B_1 + B_2 x$

if x change by 1 unit, y will change $B_2 \times 100$ %

analisis

$$\text{Outp} = 0.4278 + 0.0031 \text{ age}$$

$\ln y$ $\ln x$

① analisis persamaan persamaan linear

if age increases by 1 year, outp will increase by 0.031 time

$\therefore$ Base on my own experience, when we grow older we will be more vulnerable. So, we need to go to the hospital more often. I think that this equation sentence is make sense. $\times$

c) If $outp_i$ is turned into natural logarithmic scale ($\ln$), how would you reinterpret the relationship between β_2 and $outp_i$, assumed that the given coefficient given in the table above can be used to interpret this new functional form.

Theory $\ln$ $\ln$ $\ln$

①. linear $y = B_1 + B_2 x + u$

if x change by 1 unit y will change $= B_2$ Unit

② Double log : $\ln y = B_1 + B_2 \ln x + u$

if x change by 1%, y will change $= B_2$ %

③ Linear log : $y = B_1 + B_2 \ln x$

if x change by 1%, y will change $B_2 / 100$ unit

④ log - lin $\ln y = B_1 + B_2 x$

if x change by 1 unit, y will change $B_2 \times 100$ %

Example

$outp = 0.4278 + 0.0031 age$

$\ln outp = 0.4278 + 0.0031 age$

if age increase by 1 year, outp will increase by 0.0031×100 %
 $= 0.31$ %

d) If age_i variable is divided by 10, how does it affect both the coefficients, standard errors, and confidence intervals? Answer: the changes of both the constant and slope (if there is).

$outp = B_1 + B_2 age + u$

$y = B_1 + B_2 x + u$

$$\hat{B}_2 = \frac{\sum [x - \bar{x}] [y - \bar{y}]}{\sum [x - \bar{x}]^2}$$

$$\hat{B}_1 = \bar{y} - \hat{b}_2 x$$

Standard error $SE = \frac{SD}{\sqrt{N}}$ $SD = \sqrt{Var}$

$Var(B_2) = \frac{\sigma^2}{\sum [x - \bar{x}]^2}$ $Var(B_1) = \frac{\sigma^2 \sum x^2}{n \sum [x - \bar{x}]^2}$

d). $\hat{\alpha}_{OLS} = \hat{\alpha}_{OLS} / 10$ [qin qno']

$$\hat{\beta}_2 \hat{\alpha}_{OLS} = \frac{\sum [x'' - \bar{x}''] [y - \bar{y}]}{\sum [x'' - \bar{x}']^2}$$

$$\hat{\beta}_2 \hat{\alpha}_{OLS} = \frac{\sum [\frac{x}{10} - \frac{\bar{x}}{10}] [y - \bar{y}]}{\sum [\frac{x}{10} - \frac{\bar{x}}{10}]^2} = \frac{\frac{1}{10} \sum [x - \bar{x}] [y - \bar{y}]}{[\frac{1}{10}]^2 \sum [x - \bar{x}]^2}$$

$$\hat{\beta}_2 \hat{\alpha}_{OLS} = \frac{\sum [x'' - \bar{x}''] [y - \bar{y}]}{[\frac{1}{10}] \sum [x - \bar{x}]^2} \\ = \frac{1}{0.1} \left[\frac{\sum (x - \bar{x}) (y - \bar{y})}{\sum (x - \bar{x})^2} \right]$$

$$\hat{\beta}_2 \hat{\alpha}_{OLS} = \frac{\hat{\beta}_{OLS}}{0.1} = \frac{0.0031}{0.1} = 0.031 \neq$$

$$\hat{\beta}_{1,N} = \bar{y} - \hat{\beta}_2 \bar{x}$$

$$\hat{\beta}_{1,N} = \bar{y} - \left[\frac{\hat{\beta}_2}{0.1} \right] \left[\frac{\bar{x}}{10} \right]$$

$$\hat{\beta}_{1,N} = \bar{y} - \hat{\beta}_2 \bar{x}$$

$$\hat{\beta}_1 = \hat{\beta}_{1,N} = 0.4272 \neq$$

e) $\hat{y}_0 = 0.428 + 0.0031(50) = 0.583$

$$\text{from } se(\hat{y}_0) = \sqrt{\text{var}(\hat{y}_0)} \\ = 0.0045$$

$$\text{significant level} = 0.01/2 = 0.005$$

$$t_{0.005, 9} = 2.576$$

$$\Pr(0.583 - (2.576)(0.0045) \leq y_0 \leq 0.583 + (2.576)(0.0045)) \\ = 0.99$$

$$\Pr(0.5714 \leq y_0 \leq 0.5946) = 0.99 \neq$$

Question 3

$$\text{Var}(\hat{y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]$$

if $x_0 - \bar{x}$ gets larger, the larger of variance leading to larger $\text{se}(\hat{y}_0)$

$\bar{x}$ is the central tendency of x_i , the less information of x_i is available. Since, the Confidence interval must be larger to get the more unknown value.