

# EE 320 Introductory Mathematical Economics

Semester 2/2012

## Problem Set 5

### Differential Calculus and Applications (1 independent variable)

1. Exercise 7.2 questions 1, 2, 4, and 10 in Chiang and Wainwright (2005)
2. Exercise 9.2 question 1 in Chiang and Wainwright (2005)
3. Exercise 9.4 question 3 in Chiang and Wainwright (2005)

*(Questions 4 – 7 are from Sydsaeter and Hammond, 2008)*

4. Suppose that

$$R(Q) = 10Q - \frac{Q^2}{1000}, \quad C(Q) = 5000 + 2Q, \quad Q \in [0, 10,000]$$

Find the value of  $Q$  that maximizes profits.

5. The price a firm obtains for a commodity varies with demand  $Q$  according to the formula  $P(Q) = 18 - 0.006Q$ . Total cost is  $C(Q) = 0.004Q^2 + 4Q + 4500$ .

- a. Find the firm's profit and the value  $Q$  which maximizes profit.
- b. Find a formula for the elasticity of  $P(Q)$  w.r.t.  $Q$ , and find the particular value  $Q^*$  of  $Q$  at which the elasticity is equal to -1.
- c. Show that the marginal revenue is 0 at  $Q^*$ .

6. Find the local extreme points for:

$$\text{a) } f(x) = x^3 e^x \qquad \text{b) } g(x) = x^2 2^x$$

7. A competitive firm receives a price  $p$  for each unit of its output, and pays a price  $w$  for each unit of its only variable input. It also incurs set up costs of  $F$ . Its output from using  $x$  units of variable input is  $f(x) = \sqrt{x}$ . Determine the firm's revenue, cost, and profit functions.

*(Questions 8 – 10 are from Dowling, 2012)*

8. Find the marginal and the average functions for each of the following total functions.

$$\begin{aligned} \text{a) } TC &= 3Q^2 + 7Q + 12 \\ \text{b) } \pi &= Q^2 - 13Q + 78 \end{aligned}$$

9. For the following functions, (i) find the critical values, (ii) test for concavity to determine relative maxima or minima, (iii) check for inflection points, and (4) evaluate the function at the critical value and inflection points.

a)  $f(x) = x^3 - 18x^2 + 96x - 80$

b)  $f(x) = -x^3 + 6x^2 + 15x - 32$

c)  $f(x) = (2x - 7)^3$

10. Given  $C = 1200 + 0.8Y_d$ ,  $Y_d = Y - T$ , and  $T = 100$ , use the derivative find the marginal propensity to consume (MPC).