Autocorrelation

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3. Detecting Autocorrelation
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Nature of the Problem

Correlation between members of series of observations ordered in time.

Assumption of OLS:

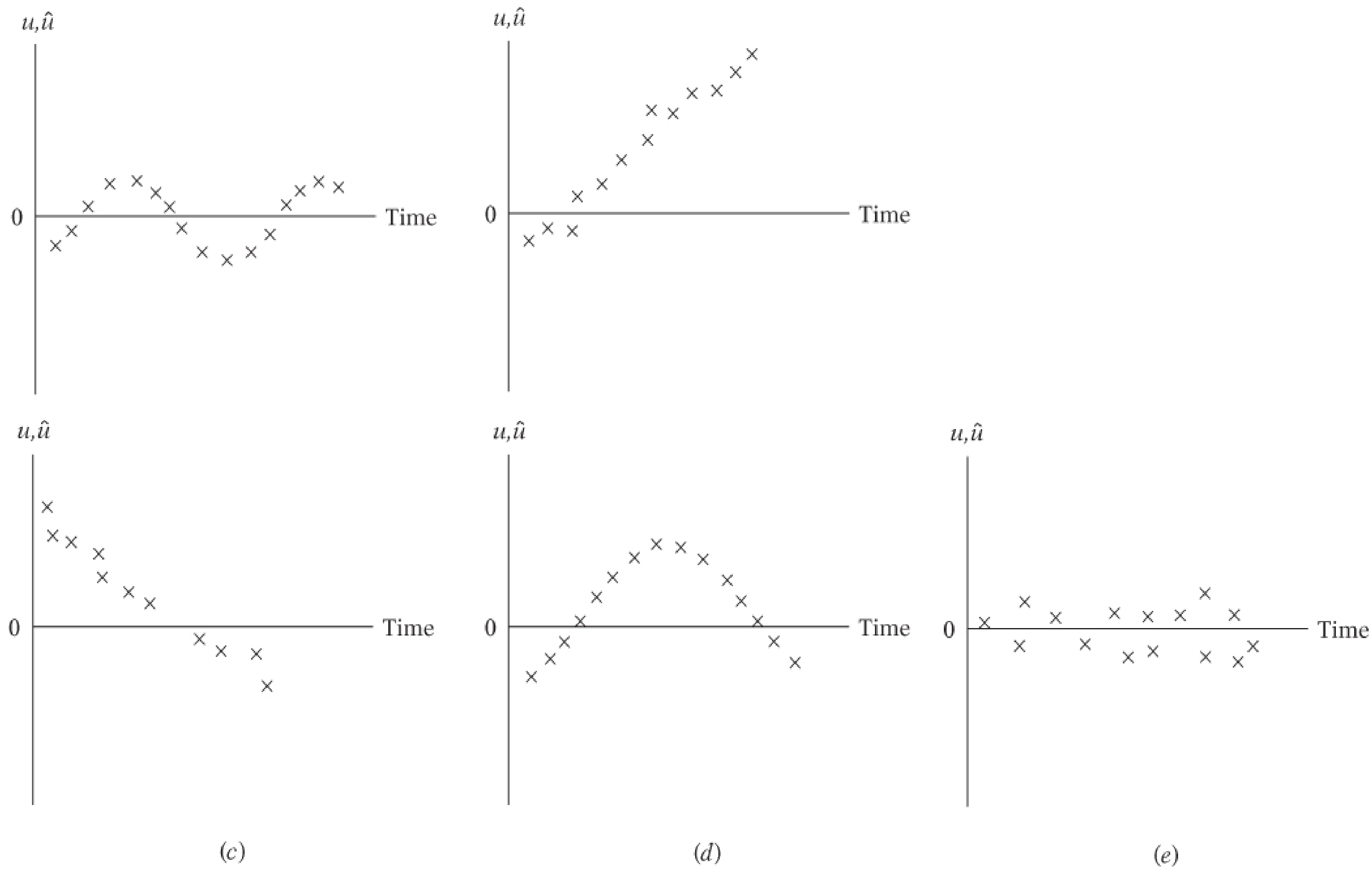
Nonautocorrelation $E(u_t u_s) = 0$ for all $t \neq s$

- No serial correlation among the disturbance terms.

Autocorrelation occurs when this assumption is violated.

Autocorrelation $E(u_t u_s) \neq 0$ for all $t \neq s$

Nature of the Problem



OLS Estimation in Presence of Autocorrelation

Let $u_t = \rho u_{t-1} + \varepsilon_t$

Two-variable Model

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

Variance of the OLS estimator of β_2 will be:

Nonautocorrelation

$$\operatorname{var}\left(\hat{\beta}_2\right) = \frac{\sigma^2}{\sum x_t^2}$$

OLS Estimation in Presence of Autocorrelation

Autocorrelation

$$\operatorname{var}\left(\hat{\beta}_2\right)_{AR1} = \frac{\sigma^2}{\sum x_t^2} + \frac{2\sigma^2}{\sum x_t^2}\left[A\right]$$

$$A = \left[\rho \frac{\sum_{t=1}^{n-1} x_t x_{t+1}}{\sum_{t=1}^{n} x_t^2} + \rho^2 \frac{\sum_{t=1}^{n-2} x_t x_{t+2}}{\sum_{t=1}^{n} x_t^2} + \cdots + \rho^{n-1} \frac{x_1 x_n}{\sum_{t=1}^{n} x_t^2}\right]$$

OLS Estimation in Presence of Autocorrelation

With autocorrelation, will OLS estimators still be BLUE?

- Best — No
- Linear — Yes
- Unbiased — Yes

BLUE Estimator in Presence of Autocorrelation

$$\hat{\beta}_2^{GLS} = \frac{\sum_{t=2}^{n}(x_t - \rho x_{t-1})(y_t - \rho y_{t-1})}{\sum_{t=2}^{n}(x_t - \rho x_{t-1})^2} + C$$

$$\operatorname{var}\left(\hat{\beta}_2^{GLS}\right) = \frac{\sigma^2}{\sum_{t=2}^{n}(x_t - \rho x_{t-1})^2} + D$$

where: C and D are correction factors which may be disregarded in practice.

Consequences of Using OLS in Presence of Autocorrelation

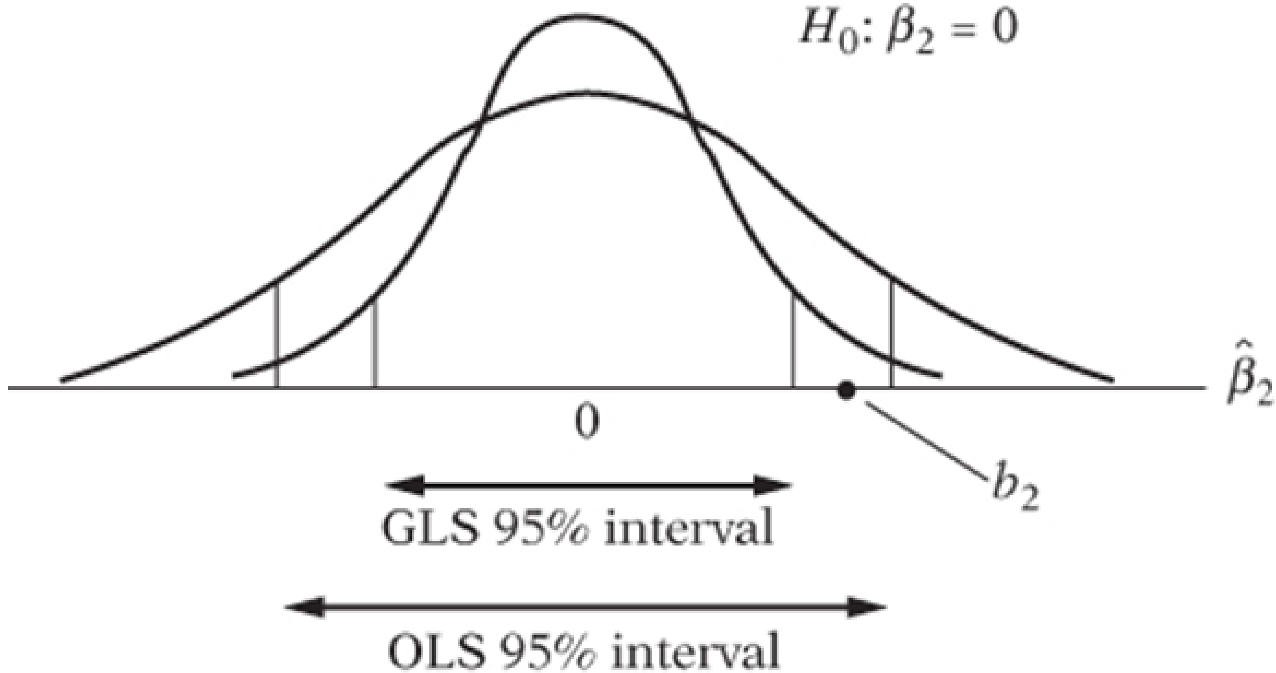
OLS Estimation allowing Autocorrelation

Since $$\operatorname{var}\left(\hat{\beta}_2^{GLS}\right) \leq \operatorname{var}\left(\hat{\beta}_2\right)$$

- Variances of estimated coefficients are more likely to be larger than they should be.
- t-test and F-test will be smaller than what is appropriate.
- Thus, t-test and F-test are more likely to give inaccurate results.

Consequences of Using OLS in Presence of Autocorrelation

To establish confidence intervals and to test hypotheses, one should use GLS and not OLS even though the estimators derived from the latter are unbiased and consistent.



Consequences of Using OLS in Presence of Autocorrelation

OLS Estimation disregarding Autocorrelation

- Variances of estimated coefficients are biased (underestimate) estimators.
- Overestimate R^2
- Conventional t-test and F-test will be inappropriated.

Detecting Autocorrelation

<u>Informal Methods</u>

- Graphical Method

<u>Formal Methods</u>

- Runs (Geary) test
- Durbin-Watson d test
- Breusch-Godfrey (BG) test

Durbin-Watson d Test

$$d = \frac{\sum_{t=2}^{t=n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=2}^{t=n} \hat{u}_t^2}$$

<u>Assumptions</u>

1. Regression model includes intercept term.
2. X's are nonstochastic.
3. Disturbances u_t – 1^{st} order autoregressive.
4. No lagged value of dependent variable.
5. No missing observations in the data.

Durbin-Watson d Test

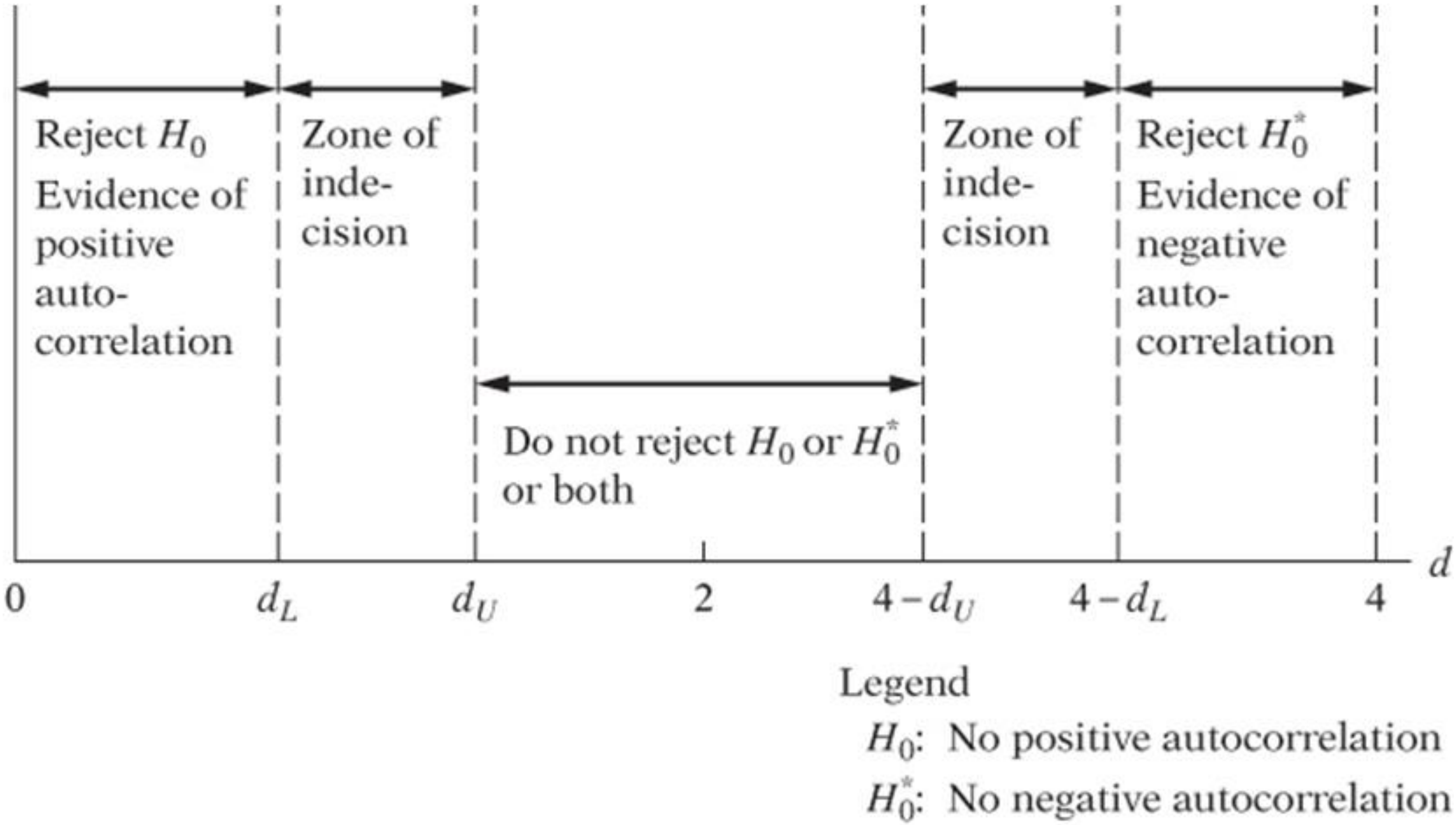
$$d = \frac{\sum \hat{u}_t^2 + \sum \hat{u}_{t-1}^2 - 2\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

$$d = 2\left(1 - \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}\right) = 2(1 - \hat{\rho})$$

$$\hat{\rho} = \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

$$-1 \le \rho \le 1 \rightarrow 0 \le d \le 4 \quad \text{if } \hat{\rho} = 0 \rightarrow d = 2$$

Durbin-Watson *d* Test



Durbin-Watson *d* Test

	$k' = 1$		$k' = 2$		$k' = 3$		$k' = 4$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.610	1.400	—	—	—	—	—	—
7	0.700	1.356	0.467	1.896	—	—	—	—
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414
11	0.927	1.324	0.658	1.604	0.595	1.928	0.444	2.283
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977

Durbin-Watson *d* Test

Null Hypothesis	Decision	If
No positive autocorrelation	Reject	$0 < d < d_L$
No positive autocorrelation	No decision	$d_L \le d \le d_U$
No negative correlation	Reject	$4 - d_L < d < 4$
No negative correlation	No decision	$4 - d_U \le d \le 4 - d_L$
No autocorrelation, positive or negative	Do not reject	$d_U < d < 4 - d_U$

Breusch-Godfrey (BG) Test

Test of higher-order autocorrelation

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \cdots + \rho_p u_{t-p} + \varepsilon_t$$

1. Estimate model using OLS and obtain estimated u_t
2. Regress the above model using estimated u_t
3. Compute $(n-p)R^2 \sim \chi_p^2$

   p = number of lagged values.

If the compute value exceeds the critical *chi-squares* value, we reject the null hypothesis of nonautocorrelation.

Remedial Measures

When structural of autocorrelation is known

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1}$$

$$\left(Y_t - \rho Y_{t-1}\right) = \beta_1 \left(1 - \rho\right) + \beta_2 \left(X_t - \rho X_{t-1}\right) + \varepsilon_t$$

$$Y_t^* = \beta_1^* + \beta_2^* X_t^* + \varepsilon_t$$

Remedial Measures

When structural of autocorrelation is unknown

Estimated ρ must be a consistent estimator.

Suggested methods include:

- First difference method
- ρ based on Durbin-Watson d statistic
- Cochrane-Orcutt iterative procedure

Cochrane-Orcutt Iterative Procedure

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

1. Estimate model using OLS and obtain $\hat{u}_t$
2. Estimate $\hat{u}_t = \hat{\rho}\hat{u}_{t-1} + v_t$
3. $\left(Y_t - \hat{\rho}Y_{t-1}\right) = \beta_1\left(1-\hat{\rho}\right) + \beta_2\left(X_t - \hat{\rho}X_{t-1}\right) + \varepsilon_t$

$$Y_t^* = \beta_1^* + \beta_2^* X_t^* + \varepsilon_t^*$$

4. Iterative procedure to estimate $\hat{\rho}$