Solution

EE 325 HW. 5 #

① From

$$y_i = B_1 + B_2 x_i + U_i$$

First Alternative:

$$\hat{B}_1 = \bar{y}_i - \hat{B}_2 \bar{x}_i$$

Since $\bar{y}_i = \frac{\Sigma y_i}{n} = \frac{\Sigma [Y_i - \bar{Y}]}{n}$

$$= \frac{\Sigma Y_i}{n} - \frac{\Sigma \bar{Y}}{n}$$

$$= \bar{Y} - \bar{Y} = 0$$

Also, $\bar{x}_i = 0$ as well.

Thus, $\hat{B}_1 = 0$ in this case, or the regression line must pass through the origin.

Second Alternative:

We know that

$$y_i = \hat{B}_1 + \hat{B}_2 x_i + \hat{u}_i$$

$$\Sigma y_i = \Sigma \hat{B}_1 + \hat{B}_2 \Sigma x_i + \Sigma \hat{u}_i$$

$$\therefore \; n\hat{B}_1 = 0$$

$$\hat{B}_1 = 0$$

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Since we know that

$\Sigma y_i = \Sigma x_i = 0$

(why ??)

and $\Sigma \hat{u}_i = 0$

from the normal equation.

2:

Consider the following models:

$$\ln Y_i^* = \alpha_1 + \alpha_2 \ln X_i^* + U_i^* \quad -(1)$$

$$\boxed{\ln Y_i} = \beta_1 + \beta_2 \boxed{\ln X_i} + U_i \quad -(2)$$

where $Y_i^* = w_1 Y_i$ and $X_i^* = w_2 X_i$

From (1), we can rewrite the equation as:

$$\ln(w_1 Y_i) = \alpha_1 + \alpha_2 \ln(w_2 X_i) + u_i^*$$

$$\ln(w_1) + \ln(Y_i) = \alpha_1 + \alpha_2 \ln(w_2) + \alpha_2 \ln(X_i) + U_i^*$$

$$\boxed{\ln(Y_i)} = [\alpha_1 + \alpha_2 \ln(w_2) - \ln(w_1)] + \alpha_2 \boxed{\ln(X_i)} + u_i^*$$

Comparing this with the model (2), you will see that except for the intercept terms, the two models are the same

Therefore, the estimated slope coefficients in the two models will be the same, the only difference being in the estimated intercepts.

om model 2

Define the deviation form:

$$\overline{\ln Y_i} = \frac{\sum \ln Y_i}{n}$$

$$\overline{\ln X_i} = \frac{\sum \ln X_i}{n}$$

Let $a_i = \ln X_i - \overline{\ln X_i}$

$b_i = \ln Y_i - \overline{\ln Y_i}$

From model 1

$$\overline{\ln Y_i^*} = \frac{\sum [\ln Y_i + \ln w_1]}{n} = \frac{\sum \ln Y_i}{n} + \ln w_1$$

$$= \overline{\ln Y_i} + \ln w_1$$

$$\overline{\ln X_i^*} = \frac{\sum [\ln X_i + \ln w_2]}{n} = \frac{\sum \ln X_i}{n} + \ln w_2$$

$$= \overline{\ln X_i} + \ln w_2$$

Let $c_i = \ln X_i^* - \overline{\ln X_i^*}$

$c_i = \ln X_i + \cancel{\ln} w_2 - \ln \overline{X_i} - \cancel{\ln} w_2$

$c_i = \ln X_i - \overline{\ln X_i} = a_i$

We can also show that

$d_i = \ln Y_i^* - \overline{\ln Y_i^*} = \ln Y_i - \overline{\ln Y_i} = b_i$

for $var(\hat{\beta}_2) = \dfrac{\sigma^2}{\sum a_i^2}$

$$var(\hat{\beta}_1) = \frac{\sum(\ln x_i)^2}{n \sum a_i^2} \cdot \sigma^2$$

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

for $var(\hat{\alpha}_1) = \dfrac{\sum(\ln x_i^*)^2 \cdot \sigma^{*2}}{n \sum c_i^2}$

where $\hat{\sigma}^{*2} = \dfrac{\sum \hat{u}_i^{*2}}{n-2} = \hat{\sigma}^2$

$$= \frac{\sum[\ln x_i + \ln w_2]^2 \cdot \sigma^{*2}}{n \sum c_i^2}$$

$$= \frac{\sum[(\ln x_i)^2 + 2\ln x_i \ln w_2 + (\ln w_2)^2] \cdot \sigma^{*2}}{n \sum c_i^2}$$

$$= \frac{\sum(\ln x_i)^2 \cdot \sigma^{*2}}{n \sum c_i^2} + \frac{[2\sum \ln x_i \ln w_2 + \sum(\ln w_2)^2] \cdot \sigma^{*2}}{n \cdot \sum c_i^2}$$

$$var(\hat{\alpha}_1) = var(\hat{\beta}_1) + \frac{[2\sum \ln x_i \ln w_2 + \sum(\ln w_2)^2] \cdot \sigma^{*2}}{n \cdot \sum c_i^2}$$

We can also show that

$$var(\hat{\beta}_2) = var(\hat{\alpha}_2)$$

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For r^2 from model 2

$$r^2 = \frac{\hat{\beta}_2^2 \Sigma a_i^2}{\Sigma b_i^2}$$

For r^2 from model 3

$$r^2 = \frac{\hat{\alpha}_2^2 \Sigma c_i^2}{\Sigma d_i^2}$$

since $\hat{\beta}_2 = \hat{\alpha}_2$ and $a_i = c_i$, $b_i = d_i$

$\therefore$ the r^2 from these two models are the same #.

$\hat{\alpha}_2 = 3.5 \qquad \hat{\alpha}_1 = -3$

$\hat{\lambda}_3 = -1.357 \qquad \hat{\lambda}_1 = 4$

$\hat{B}_1 = 2 \qquad \hat{B}_2 = 1 \qquad \hat{B}_3 = -1$

$\therefore \hat{\alpha}_2 \neq \hat{B}_2$ and $\hat{\lambda}_3 \neq \hat{B}_3$

For $\hat{\alpha}_2 \neq \hat{B}_2$

since $\hat{B}_2$ takes X_{3i} into account in the regression model, $\hat{B}_2$ is used to measure the change in the mean value of Y, E[Y] per unit change in X_2, holding the value of X_3 constant.

However, $\hat{\alpha}_2$ measures the change in the mean value of Y, E[Y] per unit change in X_2 solely.

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For $\hat{\lambda}_3 \neq \hat{B}_3$

$\Rightarrow$ Same reason.

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